RAINBOWS IN THE HYPERCUBE

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Abstract. Let Q_n be a hypercube of dimension n, that is, a graph whose vertices are binary n-tuples and two vertices are adjacent iff the corresponding n-tuples differ in exactly one position. An edge coloring of a graph H is called rainbow if no two edges of H have the same color. Let $f(G, H)$ be the largest number of colors such that there exists an edge coloring of G with $f(G, H)$ colors such that no subgraph isomorphic to H is rainbow. In this paper we start the investigation of this anti-Ramsey problem by providing bounds on $f(Q_n, Q_k)$ which are asymptotically tight for $k = 2$ and by giving some exact results.

1. Introduction

We consider finite and simple graphs. An edge coloring $c : E(G) \rightarrow \{1, 2, \ldots\}$ of a graph $G = (V(G), E(G))$ is called *rainbow* if no two edges of G have the same color, that is, in a rainbow coloring the edges are totally multicolored. Given a host graph G and a subgraph $H \subseteq G$, an edge coloring is called *H-anti-Ramsey* iff every copy of H in G has at least two edges of the same color. Denote by $f(G, H)$ the maximum number of colors such that there is no rainbow copy of H in some edge coloring of G with $f(G, H)$ colors (which is of course the maximum number of colors to be used in an H-anti-Ramsey coloring of G). Equivalently, any edge coloring of G with at least $rb(G, H) = f(G, H) + 1$ colors contains a rainbow copy of H. The number $rb(G, H)$ will be called *rainbow number* and $f(G, H)$ *anti-Ramsey number* of graphs G and H.

The function $f(G, H)$ was introduced by Erdős, Simonovits and Sós [4]. They determined the asymptotic behavior of $f(K_n, H)$ for graphs H such that the deletion of any edge results in a graph with chromatic number at least 3 showing that this function is closely related to the extremal function $ex(n, H)$ which gives the maximum number of edges in a subgraph of K_n containing no copy of a graph H. See also [7] for exact results.

Anti-Ramsey problems drew a lot of attention in the past few years. Most of them are investigating $f(K_n, H)$, that is, when the edge-colored graph is complete (see, e.g., [6], [8], [9], [10]). There is a number of results on anti-Ramsey numbers under different local constraints and on anti-Ramsey numbers of the structures other than graphs, such as posets, integers and so on.

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One of the less studied directions in this area is investigation of the anti-Ramsey function $f(G, H)$ when G is not complete. It has been studied for G being complete bipartite [1], [2]. The anti-Ramsey properties of Ramanujan graphs were investigated in [5]. In this note, we start the investigation of the anti-Ramsey numbers for hypercubes G and H.

We give some basic definitions concerning hypercubes first. For all other general graph-theoretic notations and definitions, we refer the reader to [11]. We assign all (0,1)-sequences of length n to the 2^n vertices of Q_n such that two vertices are adjacent if and only if the assigned sequences differ in exactly one position. We consider a drawing of Q_n such that all vertices with the same number of 1's in the assigned sequence are drawn in one row (such a drawing is called layer model or Hasse diagram of Q_n). So we get $n+1$ vertex layers $V_0, V_1, \dots, V_n$ and n edge layers $E_1, E_2, \dots, E_n$ in between consecutive V_is.

2. Bounds for $f(Q_n, Q_k)$

In this section we give general lower and upper bounds for $f(Q_n, Q_k)$.

Theorem 1.

$$f(Q_n, Q_k) \geq n\, 2^{n-1} - \left\lfloor \frac{n}{k} \left(2^{n-1} - k + 1\right) \right\rfloor .$$

Proof: Let $L = L(i, k)$ be a subgraph of Q_n formed by all edge layers E_t of the Hasse diagram, where $t \equiv i \pmod{k}$, that is, L consists of every kth edge layer of Q_n. Consider a coloring of $E(Q_n)$ such that at most $k-1$ colors are used on each layer in L and new distinct colors used on all other edges of Q_n. Such a coloring has at least $n\, 2^{n-1} - \lfloor \frac{n}{k}(2^{n-1} - k + 1) \rfloor$ colors since the average $(n\, 2^{n-1} - n(k-1))/k$ of the maximum numbers of equally colored edges in the edge layers of Q_k is subtracted from the number $n\, 2^{n-1}$ of edges of Q_n. Any subcube Q_k contains at least k edges from some layer of L which implies that there is at least one pair of edges of Q_k having the same color in that layer. □

Setting $k = 2$ in Theorem 1 we immediately obtain the following result.

Corollary 1.

$$f(Q_n, Q_2) \geq n\, 2^{n-2} + \left\lceil \frac{n}{2} \right\rceil .$$

A very easy upper bound $f(Q_n, Q_2) \leq (0.623 + o(1)) n 2^{n-1}$ can be obtained from the result of Chung [3] on dense subgraphs of a hypercube not containing C_4 (a cycle of length 4). Indeed, consider an edge-coloring of Q_n with no rainbow Q_2s. Then, each representing graph consisting of edges having distinct colors does not contain a $Q_2 = C_4$. Therefore, the number of edges in such a graph [3] which is equal to the number of colors used is at most $(0.623 + o(1)) n 2^{n-1}$.

We can prove a general upper bound which matches our lower bound for $k = 2$ asymptotically.

Theorem 2.

$$f(Q_n, Q_k) \leq n\, 2^{n-1} \left(1 - \frac{n-k}{(n-1) k 2^{k-2}} \right).$$

Proof: Consider a Q_k-anti-Ramsey coloring of Q_n. For any subgraph G_i of Q_n determined by all $i+1$ edges of one color ($i \geq 1$) let q_i be the number of nonrainbow Q_k's having at least 2 edges from the color of G_i. There may be different subgraphs G_i for an i. Every pair of the edges of a G_i occurs in at most $\binom{n-2}{k-2}$ of the Q_ks so that

$$q_i \leq \binom{i+1}{2} \binom{n-2}{k-2}. \tag{1}$$

On the other hand, every edge of a G_i is an edge of $\binom{n-1}{k-1}$ different Q_ks in Q_n and every Q_k is counted for at least two of the $i+1$ edges so that

$$q_i \le \frac{i+1}{2}\binom{n-1}{k-1}. \tag{2}$$

The bound (1) is less than that of (2) for $i < (n-1)/(k-1)$.

For any partition of the integer $n2^{n-1} - f(Q_n, Q_k) = n2^{n-1} - f = \sum i$ into summands i let $\sum q_i$ denote the sum of all values q_i corresponding to the summands i. The assumed Q_k-anti-Ramsey coloring determines one partition of $n2^{n-1} - f$ with summands i for each of the G_is, that is, for each color with $i+1$ edges. It is necessary that $\sum q_i$ for this partition and thus also the largest of the sums $\sum q_i$ over all possible partitions of $n2^{n-1} - f$ is at least $\binom{n}{k}2^{n-k}$ being the number of Q_ks in Q_n.

Let $s = \lceil (n-1)/(k-1) \rceil - 1$. For a partition with summands i and j, $i < s$ and $i \le j$ we obtain with (1) and (2)

$$q_i + q_j \le \binom{i+1}{2}\binom{n-2}{k-2} + \frac{j+1}{2}\binom{n-1}{k-1} \le \frac{i+j+1}{2}\binom{n-1}{k-1} \tag{3}$$

so that with (1) and (2) the upper bound of $\sum q_i$ for this partition is not decreased if with (2) the upper bound of $\sum q_i$ is considered for that partition having a summand $i+j$ instead of the two summands i and j. Thus we can restrict ourselves to all partitions with summands $i \ge s$ only. For these partitions the number p of summands is a maximum if there are as many summands s as possible, that is, $p \le (n2^{n-1} - f)/s$. Altogether, using (2) we obtain an upper bound of the sums $\sum q_i$ for all partitions and thus the necessary condition

$$\begin{aligned} \binom{n}{k}2^{n-k} &\le \sum q_i \le \sum \frac{i+1}{2}\binom{n-1}{k-1} = \frac{1}{2}\binom{n-1}{k-1}\left(n2^{n-1} - f + p\right) \\ &\le \frac{1}{2}\binom{n-1}{k-1}\left(n2^{n-1} - f\right)\left(1 + \frac{1}{s}\right) \end{aligned} \tag{4}$$

implying

$$f \le n2^{n-1}\left(1 - \frac{1}{k2^{k-2}(1+1/s)}\right) \le n2^{n-1}\left(1 - \frac{n-k}{(n-1)k2^{k-2}}\right).$$

□

For fixed k, the lower and upper bounds on $f(Q_n, Q_k)/(n2^{n-1})$ by Theorem 1 and Theorem 2 go to $1 - \frac{1}{k}$ and $1 - \frac{1}{k2^{k-2}}$, respectively, as $n \to \infty$.

Corollary 2. *If $(n-1)/(k-1)$ is an integer then*

$$f(Q_n, Q_k) \le n\,2^{n-1}\left(1 - \frac{n-1}{(n+k-2)k2^{k-2}}\right).$$

Proof: In the preceding proof we can restrict ourselves because of (3) to all partitions with summands $i \ge s+1$ if $s+1 = (n-1)/(k-1)$ is an integer. Then $p \le (n2^{n-1} - f)/(s+1)$ in (4) implies

$$f \le n\,2^{n-1}\left(1 - \frac{1}{k2^{k-2}(1+1/(s+1))}\right) = n\,2^{n-1}\left(1 - \frac{n-1}{(n+k-2)k2^{k-2}}\right).$$

□

Setting $k = 2$ in Corollary 2 we obtain

Corollary 3.

$$f(Q_n, Q_2) \leq (n+1)\, 2^{n-2}.$$

3. Determining $f(Q_n, Q_{n-1})$

For $k = n-1$ we determine the exact value of $f(Q_n, Q_k)$. It turns out that all edges except three can be colored differently if $n \geq 6$.

Theorem 3. $f(Q_n, Q_{n-1}) = \begin{cases} n\, 2^{n-1} - 4 & \textit{for} \quad n = 3, 4, 5, \\ n\, 2^{n-1} - 3 & \textit{for} \quad n \geq 6. \end{cases}$

Proof: First we provide constructions of the colorings for the lower bounds.

$f \geq n\, 2^{n-1} - 4$: For both edge layers E_1 and E_n of the Hasse diagram of Q_n we choose a triple of edges and assign a color 1 to the first triple and color 2 to the second one; we color all other edges with new and distinct colors. This way every Q_{n-1} has 2 or 3 edges having the same color. There are $n\, 2^{n-1} - 4$ colors used in this coloring.

$f \geq n\, 2^{n-1} - 3$ for $n \geq 6$: Consider a Hasse diagram of Q_n such that in each layer $V_i, i = 0, \ldots, n$, all $(0,1)$-sequences of the vertices are in lexicographic order. Give the 2 rightmost edges of E_1 color 1 and the 2 rightmost edges of E_n color 2. Then only 2 Q_{n-1}s from the top V_0 of the Hasse diagram of Q_n and 2 Q_{n-1}s from the bottom V_n remain without a monochromatic pair of edges of colors 1 or 2. These 4 Q_{n-1}s contain a Q isomorphic to Q_{n-4} in their intersection. Lets assign color 3 to any two edges of Q (note that this is possible for $n \geq 6$) and color all uncolored edges with new and distinct colors. Then, each Q_{n-1} has at least one pair of edges having the same color and the total number of colors is $n2^{n-1} - 3$.

Now, we shall provide the desired upper bounds. We shall use the fact that the number of Q_{n-1}s in Q_n is $2n$.

$f \leq n\, 2^{n-1} - 3$: Assume that there are at least $n\, 2^{n-1} - 2$ colors used. First, assume that there are two distinct color classes of size 2 each. Then the edges of these color classes occur in at most $2\binom{n-2}{n-1-2} = 2(n-2) < 2n$ different Q_{n-1}s, a contradiction. Second, assume that there is a color class of size 3. One edge of Q_n is in $\binom{n-1}{n-2} = n-1$ different Q_{n-1}s. Then the 3 edges of this color class may occur in $3(n-1)$ of the Q_{n-1}s, and at most $3(n-1)/2 < 2n$ different Q_{n-1}s have some 2 of these 3 edges, a contradiction.

$f \leq n\, 2^{n-1} - 4$ for $n = 3, 4, 5$: Assume that there are at least $n\, 2^{n-1} - 3$ colors used. If there are 3 monochromatic pairs of edges, then these pairs occur in at most $3(n-2) < 2n$ different Q_{n-1}s, a contradiction for $n < 6$. A monochromatic triple of edges and a monochromatic pair of edges of another color occur together in at most $3(n-1)/2 + n - 2 < 2n$ different Q_{n-1}s, a contradiction for $n < 6$. Furthermore, 2 of 4 edges of the same color occur in at most $4(n-1)/2 < 2n$ different Q_{n-1}s, again a contradiction.

□

4. Determining $f(Q_4, Q_2)$

From Corollary 1 and Corollary 3 in Section 2 we obtain $18 \leq f(Q_4, Q_2) \leq 20$. We prove that the lower bound is attained.

Theorem 4. $f(Q_4, Q_2) = 18$.

Proof: Setting $n = 4$ in Corollary 1 we obtain $f(Q_4, Q_2) \geq 18$.

For the upper bound we present two proofs which use totally different methods.

Proof 1 of $\mathbf{f(Q_4, Q_2) \leq 18}$: Let c be a coloring of $E(Q_4)$ with no rainbow C_4 using at least 19 colors. Let $c(E)$ denote the set of colors used in an edge set E.

Claim 1. There is a pair Q, Q' of vertex disjoint Q_3s in Q_4 using together at least 15 colors on their edges.

Assume that every pair of vertex disjoint Q_3s uses at most 14 colors. Since Q_4 contains 4 pairs of vertex disjoint Q_3s and every edge is counted in $\binom{3}{2}$ Q_3s, the total number of colors is at most $14 \cdot 4/3 < 19$, a contradiction.

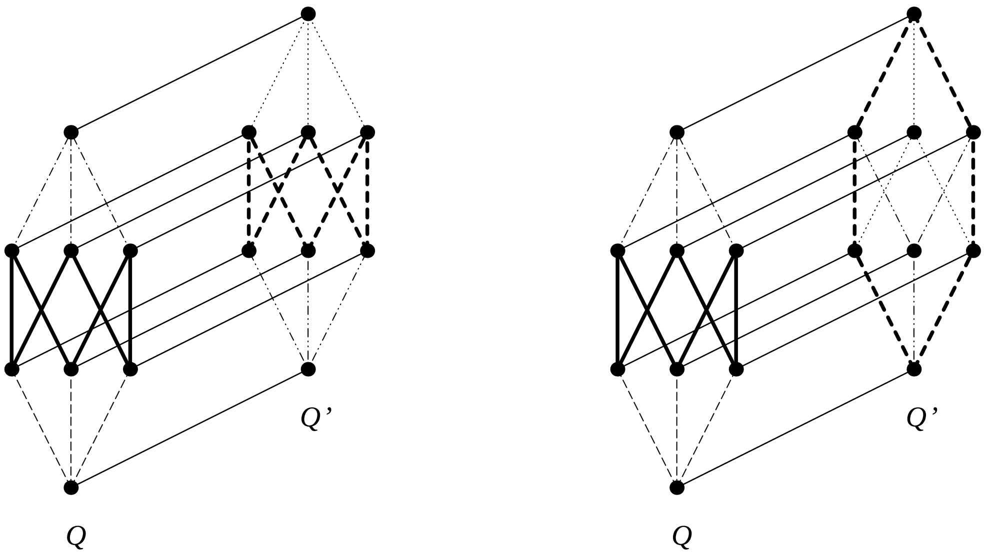

FIGURE 1. Bold edges form a rainbow subgraph in each of Q and Q'.

Next, we shall assume that Q uses 8 colors and Q' uses 7 or 8 colors (since $f(Q_3, Q_2) = 8$ by Theorem 3). We say that an edge from Q is parallel to an edge from Q' if the corresponding endpoints are adjacent. Only cycles C_4, C_6, and C_8 occur in Q. By assumption, there is no rainbow C_4 in Q, there is no rainbow C_6 with a chord since otherwise this forces a rainbow C_4. Moreover, there is no rainbow C_8 since otherwise two parallel chords force a rainbow C_4. Thus there is a chordless rainbow C_6 in Q (see bold edges in Figure 1) since Q has 8 vertices and 8 edges of different colors and must therefore contain a rainbow cycle. In the remaining two 3-stars of Q one edge with a color different from all colors of the C_6 forces the two other edges to be of the same color since otherwise there would be a rainbow C_4. Hence we can assume that the coloring of Q with 8 colors is unique as in Figure 1.

Let E be a set of edges between Q and Q' such that $c(E) \cap (c(Q) \cup c(Q')) = \emptyset$ and any two edges of E are of different colors. Let V be the set of endpoints of edges from E in Q. We shall analyse in three cases the possible configurations of V to arrive at contradictions. We shall need the following properties.

*P*1. If $v, u \in V$, $vu \in E(Q)$ then $c(vu) = c(v'u')$ where $v'u'$ is an edge of Q' parallel to vu.

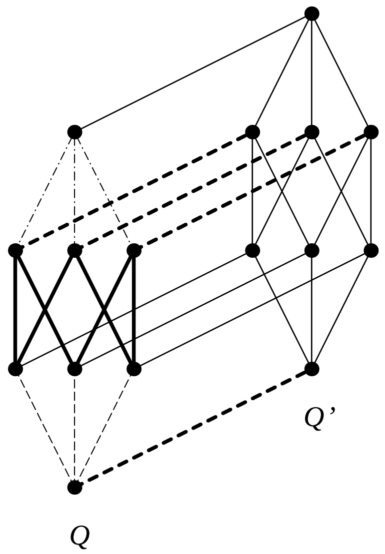

FIGURE 2. Bold dashed lines are edges of E.

$P2$. If $v, u \in V$ with $d(v, u) = 2$ in Q, that is, $vw, wu \in E(Q)$ then the edges $vw, wu, v'w', w'u'$ can use at most three colors together (here u', v', w' are vertices in Q' adjacent to u, v, w, respectively). To see that Property $P2$ holds, assume the opposite, which implies that $vwuu'w'v'v$ is a rainbow C_6, and it has a chord, a contradiction.

Case A. The number of colors in Q' is 8 and the total number of colors in Q and Q' is 16. In particular it implies that $c(Q) \cap c(Q') = \emptyset$. Note that there are only two possible colorings of Q and Q' as shown in Figure 1 since one chordless C_6 can be chosen as in Q and the chordless C_6' in Q' is parallel to C_6 or one of three nonparallel C_6's being equivalent by rotation of Q'. Moreover, $|E| \geq 3$, since the total number of colors is at least 19. Note that if $v, u \in V$ then $vu \notin E(Q)$ otherwise it will contradict $P1$. But since $|V| \geq 3$, there are two vertices $v, u \in V$ with $d(u, v) = 2$ in Q. It is clear from Figure 1, that for any two such vertices u, v there exists $w \in V(Q)$ such that the edges $uw, wv, u'w', w'v'$ use 4 colors (here u', v', w' are adjacent to u, v, w, respectively), contradicting Property $P2$.
Case B. The number of colors in Q' is 7 and the total number of colors in Q and Q' is 15. Thus the color sets used in Q and Q' are disjoint. Then $|E| \geq 4$. Again, from $P1$ we see that there are no two adjacent vertices of V. Thus V is an independent set of Q of order 4 and therefore $|E| = 4$. Thus the only possible coloring and location of E (up to isomorphism) must be as in Figure 2. In order to Property $P2$ to hold we must have all paths of length 2 in Q' which use an edge from the lower level and an edge from the middle level to be monochromatic. Thus there are at most three colors in the two lower levels of Q' and there are at most three colors in the upper level of Q'. Therefore, the total number of colors in Q' is at most 6, a contradiction.

Case C. The number of colors in Q' is 8 and the total number of colors in Q and Q' is 15. The unique color sets used in Q and Q' have one common color a. The only possible colorings are as in Figure 1. We have $|E| \geq 4$.

If $u, v \in V$ and $uv \in E(Q)$ then there is no other pair of vertices from V forming an edge in Q vertex disjoint from uv, otherwise $P1$ will not hold.

Also, if v_1, v_2, v_3, v_4 form a Q_2 in Q then there are at most two vertices of these four in V. Otherwise, consider pairs v_i, v_i', $i = 1, \dots, 4$, where v_i' is adjacent to v_i. These vertices form a Q_3 using exactly 8 colors. Thus its coloring is unique up to isomorphism having two disjoint monochromatic stars of size 3 each. The two center vertices of the 3-stars of this Q_3 have to be also center vertices in Q and Q', respectively, which is impossible.

Thus we have at most two vertices of V in each Q_2 of Q and at most one pair of adjacent vertices of V. Then the existence of one pair of adjacent vertices of V together with $|E| \geq 4$ forces a second pair disjoint from the first, a contradiction. This implies that V must be an independent set in Q and therefore $|E| = 4$. Then the contradiction follows as in Case B. □

Proof 2 of $\mathbf{f(Q_4, Q_2) \leq 18}$: We assume the existence of a Q_2-anti-Ramsey coloring of Q_4 with 19 colors, that is, with nonrainbow Q_2s only. For any subgraph G_i of Q_4 determined by all $i+1$ edges of one color ($i \geq 1$) let q_i denote the number of nonrainbow Q_2s having at least 2 edges from the color of G_i. There may be different G_is for an i. We have

$$q_1 \leq 1,\ q_2 \leq 3, \text{ and } q_i \leq 3(i+1)/2 \text{ for } i \geq 3 \tag{5}$$

since every edge in Q_4 is an edge of 3 different Q_2s and every Q_2 is counted at least twice in $3(i+1)$.

To obtain the number 19 of different colors in Q_4 we have to subtract 13 from the number 32 of edges in Q_4. Thus 13 additional edges are distributed among the 19 color classes. Then for all 24 partitions $13 = a + b + \dots$ with $a \geq b \geq \dots \geq 2$ the numbers $q_a + q_b + \dots$ of nonrainbow Q_2s are possibly attaining 24, which is the number of Q_2s in Q_4, only in the following 5 cases

(A) $q_7 + q_3 + q_3 \leq 24$,
(B) $q_5 + q_5 + q_3 \leq 24$,
(C) $q_5 + q_3 + q_3 + q_2 \leq 24$,
(D) $q_4 + q_3 + q_3 + q_3 \leq 25$,
(E) $q_3 + q_3 + q_3 + q_2 + q_2 \leq 24$.

All further partitions of 13 contain at least one term 1. Then a, or b, or $\dots$ in one of the previous partitions is diminished by 1 as far as there are 13 terms 1, and $13q_1 \leq 13 < 24$. Now any inequality $q_a + q_b + \dots < 24$ implies all inequalities $q_{a-1} + q_b + \dots < 24$, $q_a + q_{b-1} + \dots < 24$, ... For those partitions corresponding to (A) to (E) these implications follow from the differences $q_i - q_{i-1}$ for the upper bounds (5) of q_i and q_{i-1} which are $2, 2, 3, 2$ for $i = 7, 5, 3, 2$, respectively. There remains only one exception, that is, $q_4 - q_3 = 1$ in (D) yielding

(F) $q_3 + q_3 + q_3 + q_3 + q_1 \leq 25$.

If for a fixed edge all 3 different Q_2s in Q_4 are counted as nonrainbow Q_2s then an edge either adjacent or parallel to the fixed edge has to have the same color as this fixed edge. Thus Figure 3 presents all possibilities that one edge, say the leftmost edge of E_1 in Q_4, determines all 3 different Q_2s to be nonrainbow.

From Figure 3 we obtain that $q_3 = 6$ occurs only in case (a) and otherwise we have $q_3 \leq 4$ which can be seen as follows. In each of the cases (b) to (f) of Figure 3 there occur at most 4 nonrainbow Q_2s. If each edge of G_3 counts at most 2 nonrainbow Q_2s then $q_3 \leq 2(3+1)/2 = 4$. If $q_3 \leq 4$ in at least one case then we obtain 23 as an upper bound in (A) to (F). Thus we can assume $q_3 = 6$ in the following.

If $q_4 = 7$ is assumed then there exists one edge which determines exactly 2 nonrainbow Q_2s (see (5)). Deletion of this edge leaves 4 edges determining 5 nonrainbow Q_2s, a contradiction to $q_3 \neq 5$. Thus we have $q_4 \leq 6$ and therefore 24 as an upper bound in (D).

If q_i for odd i attains the upper bound of (5), that is, three nonrainbow Q_2s are counted for each edge, then G_i has 5 vertices for $i = 3$ (see case (a) of Figure 3) and at least 7 vertices for $i = 5$ and $i = 7$. This follows since at least one additional edge being nonadjacent to the edges of G_3 has to exist in case (a) of Figure 3, in cases (b) to (d) at least two additional edges have to exist to obtain 3 nonrainbow Q_2s containing an edge of E_2, and in cases (e) and (f) there are already at least 7 vertices. If there are two of these G_is for $i = 3$, 5, or 7 with a common vertex then one of the nonrainbow Q_2s is counted twice.

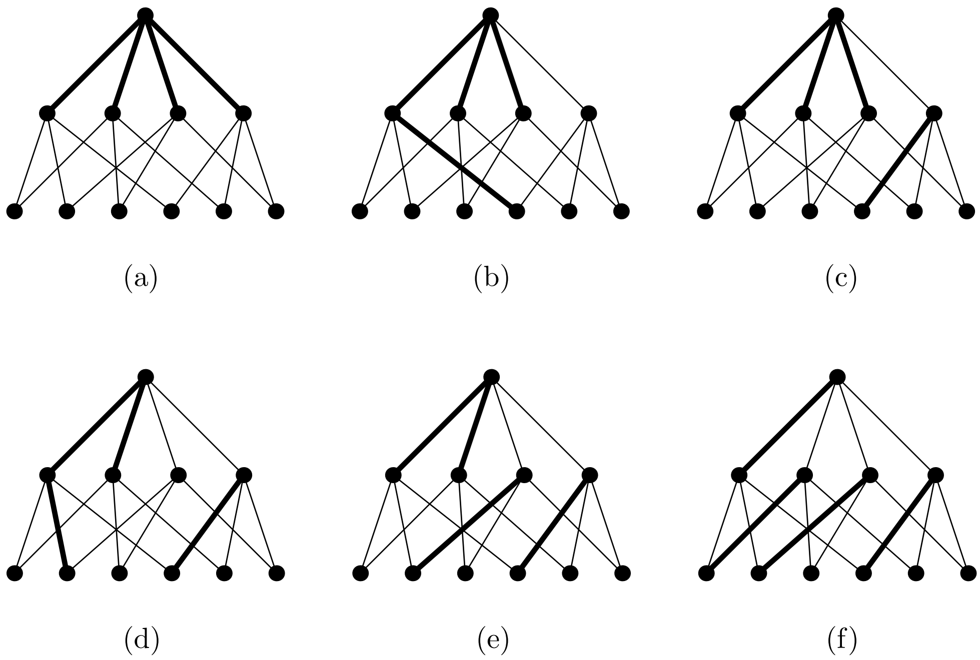

FIGURE 3. Nonrainbow cases

As a consequence we have 23 as an upper bound in (A), (B), and (C) since otherwise the sums of the numbers of vertices of the G_is are at least 17, 19, and 17, respectively, which is impossible in Q_4.

Any G_3 with $q_3 = 6$ has a vertex of degree 4 (see Figure 3, case (a)). For two vertex disjoint G_3s with $q_3 = 6$ the vertices of degree 4 have to have a distance of at least 3. Any vertex in Q_4 has distances of at least 3 to five vertices only. These five vertices have pairwise distances less than 3. Thus three vertex disjoint G_3s with $q_3 = 6$ cannot exist. Then common vertices of 1 and 2 pairs of G_3s diminish the upper bounds of $q_3 + q_3 + q_3$ and $q_3 + q_3 + q_3 + q_3$ by 1 and by 2, respectively. Thus, 23 as an upper bound is obtained for (D) to (F), too.

Altogether, in all cases we have received the contradiction that less than 24 nonrainbow Q_2s occur if 19 colors in Q_4 are assumed. □

5. Remarks

Consider a coloring of $E(Q_n)$ such that at most $k-1$ colors are used for the edges of every kth edge layer of the Hasse diagram of Q_n with different colors for different layers. All edges of the other layers are colored with distinct colors different from the colors of the kth edge layers. We conjecture that such a coloring determines the exact values of $f(Q_n, Q_k)$ for $k = 2$, that is, the lower bound of Theorem 1 is attained.

However, this is not true for $k \geq 3$. For example, for $n = 5$ and $k = 3$ we can prove $67 \leq f(Q_5, Q_3) \leq 69$. For this lower bound see Figure 4. In this coloring equally bold marked edges of Q_5 have the same color and different colors are assigned to different bold marks. All other edges are colored pairwise different and also different to the colors of the bold marked edges.

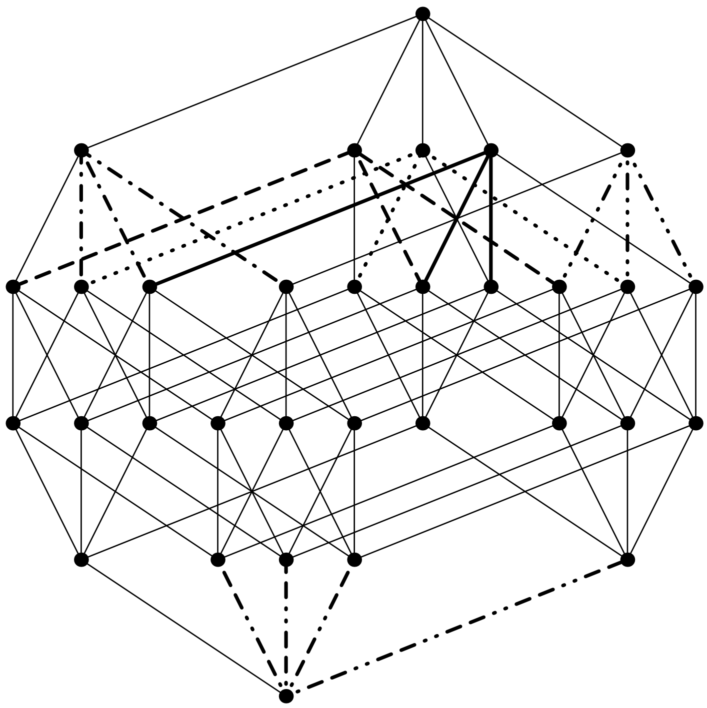

Figure 4. $f(Q_5, Q_3) \geq 67$

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