

# GEOMETRIC GROUP THEORY – SUMMER TERM 2017

ROMAN SAUER

## GENERAL INFORMATION

**Lectures.** Lectures are on Mondays, 11:30-13:00, 20.30 SR 2.058, and on Thursdays, 11:30-13:00, 20.30 SR 0.014 (with some exceptions; see the schedule below).

**The first lecture is on April 24.**

The best time to ask mathematical questions is during the lecture; the second best time is right after the lecture ends. I encourage you to ask questions in English but if you feel uncomfortable I am happy to translate your question from German.

**Exercise class.** The exercise class is on Friday, 11:30-13:00, 20.30 SR 2.058 (with some exceptions; see the schedule below) and will be given by Dr. Federico Franceschini (Office 20.30 1.020, [federico.franceschini@kit.edu](mailto:federico.franceschini@kit.edu)). **The first exercise class is on May 5.**

Federico is also the contact person if you have questions about the exercise sheets. The weekly exercise sheet will be published on Ilias on Thursday. You are supposed to submit your solutions to Federico by next Thursday (Thursday, 15:00 at the latest).

Exercises are important. Recall the famous phrase: *I hear and I forget. I see and I remember. I do and I understand.*

**Reading assignments.** For each week there is a reading assignment. You are supposed to read the assignment before Monday's lecture. You do not have to fully understand everything in the reading assignment but it will enhance your understanding of the lecture.

**Literature.** My main sources are [1] and [3]. I will cover the basics of covering theory which will be unknown to you if you have only attended *Elementare Geometrie* instead of its older version *Einführung in Geometrie und Topologie*. My background reference for covering theory is [4] – which is in German. If you need an English background reference ask us. There are many.

**Exam.** According to the Modulkatalog there will be a written exam (date yet to be decided). If there are no more than 20 students and no objections we might also do oral exams instead. Both decisions will be made by May 4 in the third lecture.

## SYLLABUS

**The following syllabus should be regarded as a rough approximation.** It will be finalized during the course. You find the current version on our website or on Ilias.

**Week 1 – April 24 and April 27.**

We roughly cover Sections 2.2 and 2.3 in [3]. Important topics are free groups and how to describe groups by generators and relations. We also present a number of constructions of new groups out of old groups.

*Reading assignment:* You need to review some basic notions about groups [3, Section 2.1] (normal subgroups, quotients etc.) and group actions [3, Section 4.1.1 up to Example 4.1.6 and Section 4.1.2 up to Example 4.1.15] (free actions, orbits etc.).

**Week 2 – May 4.**

*Note: May 1 is a holiday!*

We introduce the fundamental group of a topological space and discuss the Seifert-van Kampen theorem without proof.

*Reading assignment:* It will be easier for you if have a look at [4, Chapter 3] before.

**Week 3 – May 8 and May 11.**

We discuss the essential ideas of covering theory without formal proofs. For complete proofs you are referred to [4].

*Reading assignment:* Prepare yourself for covering theory by reading [4, Sections 4.1–4.3].

**Week 4 – May 15 and May 18.**

We roughly cover [3, Sections 3.2–3.3, 4.2–4.3]. Relationship between free groups and actions on trees. Nielsen-Schreier theorem.

*Reading assignment:* Review some issues of covering theory in greater detail by reading [4, Sections 4.4–4.6].

**Week 5 – May 22.**

*Note: May 25 is a holiday!*

We prove the ping-pong lemma and provide examples of free subgroups [3, Sections 4.3 and 4.3].

**Week 6 – May 29 and June 1.** We aim to cover quasi-isometries, word metrics and group actions as in [1, Chapter I.8]. We prove the Milnor-Schwarz lemma and provide a geometric characterization of being finitely presented. Discussion of quasi-isometry invariants.

*Reading assignment:* Have a look at [1, pp. 131–142].

**Week 7 – June 9.**

*Note: June 5 is holiday! Further, due to a conference we swap the exercise session and the Thursday lecture!*

Further discussion of quasi-isometry invariants.

*Reading assignment:* Have a look at [1, pp. 143–150].

**Week 8 – June 12.**

*Note: June 15 is a holiday!*

For the rest of the lecture we dive into the subject of hyperbolic groups. We will see what we manage to cover and the remaining syllabus will be completed in due time. Must-have topics are Dehn functions and boundaries of hyperbolic groups. See [1, Chapter III.H].

*Reading assignment:* Review what you have learned in *Elementare Geometrie* about the hyperbolic plane.

**Week 9 – June 19 and June 22.**

tba

**Week 10 – June 26 and June 30.**

*Note: Due to a conference we swap the exercise session and the Thursday lecture!*

tba

**Week 11 – July 3 and July 6.**

tba

**Week 12 – July 10 and July 13.**

tba

**Week 13 – July 17 and July 20.**

tba

**Week 14 – July 24 and 27.**

tba

## REFERENCES

- [1] M. R. Bridson and A. Haefliger, *Metric spaces of non-positive curvature*, Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 319, Springer-Verlag, Berlin, 1999.
- [2] P. de la Harpe, *Topics in geometric group theory*, Chicago Lectures in Mathematics, University of Chicago Press, Chicago, IL, 2000.
- [3] C. Löh, *Geometric group theory, an introduction*. Lecture notes, available at [www.mathematik.uni-regensburg.de/loeh/teaching/ggt.ws1415/lecture\\_notes.pdf](http://www.mathematik.uni-regensburg.de/loeh/teaching/ggt.ws1415/lecture_notes.pdf).
- [4] R. Sauer and W. Thumann, *Einführung in die Geometrie und Topologie*. Lecture notes (in German), available on Ilias.

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