

Metric Geometry

Exercise Sheet 1

You can find information about the exercise class on our homepage. If you have problems with some of the exercises, the script (especially Appendix A) might be helpful.

Exercise 1

Let X be a metric space and $A, B \subset X$ disjoint and closed. Show that there is a continuous map $f: X \rightarrow [0, 1]$ such that $f^{-1}\{0\} = A$ and $f^{-1}\{1\} = B$.

Exercise 2

Let X be a metric space. Show that X is totally bounded if and only if for all $\varepsilon > 0$ there is some compact ε -net in X .

Exercise 3

Let X be a compact metric space and $f: X \rightarrow X$ such that $|x - y| \leq |f(x) - f(y)|$ for all $x, y \in X$. Show that f is an isometry.

Exercise 4

Let X be a complete metric space. Show that X equipped with its induced length metric is also complete. (See p.16 for the definition of this metric.)