

Metric Geometry

Exercise Sheet 11

You can find information about the exercise class on our homepage. If you have problems with some of the exercises or search for further exercises, the script (especially Appendix A) might be helpful.

Exercise 1

- a) Let X be a proper CAT(0) space and $C \subset X$ closed and convex. Show that the distance function d_C to C is convex.
- b) Show that this is not true for proper CAT(1) spaces.

Exercise 2

- a) Let X be a proper length space which is locally CAT(0) and $x_0 \in X$. Show that X is CAT(0) if for every $x \in X$ there is only one geodesic connecting x with x_0 .
- b)* Let X be a compact length space which is locally CAT(0). We define the injectivity radius $ir(X)$ of X by the supremum over all $r > 0$ such that for every $x, y \in X$ with $|x - y| \leq r$ there is only one geodesic connecting x with y . Show that there is a closed local geodesic of length $2ir(X)$ if X is not CAT(0).