Metric Geometry

Exercise Sheet 4

You can find information about the exercise class on our homepage. If you have problems with some of the exercises or search for further exercises, the script (especially Appendix A) might be helpful.

Exercise 1

Let X be a complete metric tree. (See p. 14 for the definition.) Show that X is injective.

Exercise 2

a) Compute the Hausdorff distances between the following subsets of $\mathbb{R}^2$:

- $\mathbb{S}^1$
- $I := [-1,1] \times \{0\}$
- $N := \{(0,0)\}$.

b) Let $\mathbb{S}^1 \subset \mathbb{R}^2$ be equipped with its induced length metric. Construct a geodesic between the northern and southern hemisphere of $\mathbb{S}^1$ in $Haus\left(\mathbb{S}^1\right)$.

Exercise 3

Let X be a complete metric space. Show that X is a length space if and only if $Haus(X)$ is a length space.

Exercise 4

Let $\mathscr{C} \subset Haus\left(\mathbb{R}^2\right)$ be the subspace of compact convex subsets of $\mathbb{R}^2$. Show that ...

a) $\mathscr{C}$ is closed.

b) perimeter and area are continous on $\mathscr{C}$.