

Metric Geometry

Exercise Sheet 5

You can find information about the exercise class on our homepage. If you have problems with some of the exercises or search for further exercises, the script (especially Appendix A) might be helpful.

Exercise 1

Let A_r be a rectangle in the plane with sides of length 1 and r and B be a closed interval of length r . Show that $|A_r - B|_{GH} > \frac{1}{10}$ for all large r .

Exercise 2 Let $\varepsilon > 0$ and $X, Y \in GH$. Show the following statements:

- If $|X - Y|_{GH} < \varepsilon$, then there is a 2ε -isometry $f: X \rightarrow Y$.
- If there is an ε -isometry $f: X \rightarrow Y$, then $|X - Y|_{GH} < 2\varepsilon$. (*)

Exercise 3

- Let B_n be the open ball of radius $1/n$ in $\mathbb{S}^1 \subset \mathbb{R}^2$ around the north pole. Show that the sequence $\mathbb{S}^1 \setminus B_n$ is convergent in GH and determine its limit.
- Let $\mathbb{S}^1 \subset \mathbb{R}^2$ be equipped with its induced length metric and B_n be the open ball of radius $1/n$ in \mathbb{S}^1 around the north pole. Show that $\mathbb{S}^1 \setminus B_n$ equipped with its induced length metric is a convergent sequence in GH and determine its limit.

Exercise 4

Show that $\mathcal{C} := \{X \in GH : X \text{ connected}\}$ is closed in GH.