Metric Geometry

Exercise Sheet 6

You can find information about the exercise class on our homepage. If you have problems with some of the exercises or search for further exercises, the script (especially Appendix A) might be helpful.

Exercise 1

Introduction: Let X be a length space and G a group. Then G acts on X by isometries if there is a homomorphism from G to the group $Isom(X)$ of all self-isometries on X where the operation is given by composition. In this case we identify the elements of G with their images and define an equivalence relation $\sim$ on X by $x \sim y$ iff it exists $g \in G$ with $x = g(y)$. It can be shown that the metric on the corresponding quotient metric space $X/G := X/\sim$ is given by $|[x]-[y]| = \inf\{|g_1(x) - g_2(y)| : g_1, g_2 \in G\}$ for all $x, y \in X$. Also note that every subgroup of $Isom(X)$ acts on X by isometries in a natural way. (cf. [BBI, p. 76, 279 f.])

Task: Let U be a Urysohn space. Show that GH is isometric to $Haus(U)/Isom(U)$.

Exercise 2

Show that GH is geodesic.

Exercise 3 Define $A_n \subset \mathbb{R}^2$ to be the circle with radius n and center $(0,n)$. Show that A_n converges to the x-axis in the sense of Hausdorff.

Exercise 4

Compute the asymptotic cones of the following subsets of $\mathbb{R}^3$:

a) $A := \left\{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 = z\right\}$

b) $B := \left\{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 = z^2 + 1\right\}$.

References

[BBI] D. Burago, Y. Burago, S. Ivanov, *A Course in Metric Geometry*, Graduate Studies in Mathematics **33**, American Mathematical Society, 2001.