Metric Geometry

Exercise Sheet 7

You can find information about the exercise class on our homepage. If you have problems with some of the exercises or search for further exercises, the script (especially Appendix A) might be helpful.

Exercise 1

Let ω be a free ultrafilter. Show that the map

$$L\colon l^\infty(\mathbb{N}) \to \mathbb{R},$$
$$L\left((x_n)_{n\in\mathbb{N}}\right) = \lim_{n\to\omega} x_n$$

is linear and continuous.

Exercise 2

Let ω be a free ultrafilter, $(X_n)_{n\in\mathbb{N}}$ a sequence in GH and $C \subset \lim_{n\to\omega} X_n$ compact. Show that there is a subsequence $(X_{n_k})_{k\in\mathbb{N}}$ and a sequence $(E_k)_{k\in\mathbb{N}}$ with $E_k \subset X_{n_k}$ finite for every $k \in \mathbb{N}$ such that the latter sequence converges to C in GH.

Exercise 3

Let X be a proper $CAT(0)$ space and $A \subset X$ closed and convex. Show the following statements:

a) For every $x \in X$ there is a unique $p_x \in A$ such that $|x - p_x| = inf\{|x-a|\colon a \in A\}$.

b) The map $p_A\colon X \to A$, $p_A(x) = p_x$ is 1-lipschitz.

Exercise 4

We consider $l^\infty(\mathbb{N})$. Show that the angle between $\gamma_1\colon [0,1] \to l^\infty(\mathbb{N})$, $\gamma_1(t) = t(1,0,0,\ldots)$ and $\gamma_2\colon [0,1] \to l^\infty(\mathbb{N})$, $\gamma_2(t) = t(0,1,0,0,\ldots)$ in 0 does not exist.