Prof. Dr. Alexander Lytchak Summer Term 2022

Metric Geometry

Exercise Sheet 8

You can find information about the exercise class on our homepage. If you have problems with some of the exercises or search for further exercises, the script (especially Appendix A) might be helpful.

Exercise 1

Let X be a CAT(0) space and $A \subset X$ compact, connected and locally convex. Show that A is convex.
(Hint: Look at A equipped with its induced length metric.)

Exercise 2

Find a complete geodesic space which is not CAT(0) but has the property that for every triple of points there is a CAT(0) triangle having them as vertices.
(Hint: You already know such a space.)

Exercise 3

Let X be a metric space and $A \subset X$ non-empty and bounded. Show the following statements:

a) If X is proper, then there is a circumcenter of A.
(This is a point $x \in X$ with $\sup_{a\in A}|x-a| = \inf_{y\in X}\sup_{a\in A}|y-a|$.)

b) If X is CAT(0), then there is a unique circumcenter of A.
(Hint: Show that $|z-m|^2 = \frac{1}{2}\left(|z-x|^2+|z-y|^2\right) - \frac{1}{4}|x-y|^2$ for every $x,y,z \in \mathbb{R}^2$ where m denotes the midpoint between x and y.)