where

\[ \mathbb{P}(X < Y) = \frac{1}{2} \left( 1 + \frac{n}{n-1} \right) = 1 + n/n. \]

This completes the proof of the proposition.
\[
\begin{align*}
\frac{d}{dx}f(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \frac{d}{dx} f(x) \\
&= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= f'(x) \\
&= \frac{d}{dx} f(x)
\end{align*}
\]
\[
\begin{align*}
\frac{f(u)}{l} &= \frac{f'(u)}{l} = \frac{f'(u)}{\theta} = \frac{f'(u)}{\theta}
\end{align*}
\]

\[\begin{align*}
\int_{\theta}^{(\theta + \theta)} = \int_{\theta}^{(\theta + \theta)} = \int_{\theta}^{(\theta + \theta)}
\end{align*}\]
Let $g : \mathbb{R}^n \to \mathbb{R}^m$ be a function that is differentiable at a point $x \in \mathbb{R}^n$. The differential of $g$ at $x$, denoted by $dg_x$, is a linear transformation $\mathbb{R}^n \to \mathbb{R}^m$. If $h : \mathbb{R} \to \mathbb{R}^n$ is a differentiable function at $a$, then the composition $g \circ h$ is differentiable at $a$, and its derivative at $a$ is given by

$$ (g \circ h)'(a) = g'(h(a)) \cdot h'(a). $$

If $h : \mathbb{R}^n \to \mathbb{R}$ is a differentiable function, then the differential of $h$ at $x$, denoted by $dh_x$, is given by

$$ dh_x(f) = f'(x). $$

The chain rule for the composition of differentiable functions is given by

$$ (f \circ g)'(a) = f'(g(a)) \cdot g'(a), $$

where $f : \mathbb{R}^m \to \mathbb{R}^l$ and $g : \mathbb{R}^l \to \mathbb{R}^n$ are differentiable at points $a$ and $g(a)$, respectively.

If $f : \mathbb{R}^n \to \mathbb{R}$ is differentiable, then its derivative is given by

$$ df_x(f) = f'(x). $$

If $f : \mathbb{R}^n \to \mathbb{R}^m$ is differentiable, then its derivative is given by

$$ df_x(f) = f'(x). $$

If $f : \mathbb{R}^n \to \mathbb{R}^m$, then its differential at $x$, denoted by $df_x$, is a linear map $\mathbb{R}^n \to \mathbb{R}^m$. If $g : \mathbb{R}^m \to \mathbb{R}^l$, then the differential of $g$ at $y$, denoted by $dg_y$, is a linear map $\mathbb{R}^m \to \mathbb{R}^l$. The chain rule for the composition of differentiable functions is given by

$$ (g \circ f)'(a) = (g' \circ f')(a). $$

If $f : \mathbb{R}^n \to \mathbb{R}^m$ is differentiable, then its derivative is given by

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$$ (g \circ f)'(a) = (g' \circ f')(a). $$
Aufgabe 6. a) Sei $A$ das Parallelogramm mit den Eckpunkten $(0,0)$, $(1,0)$, $(0,1)$, $(1,1)$. Berechnen Sie das Integral

$$\int_A (x+y) \, dA.$$  

**Lösung:**

- Aus der Gleichung $x+y = 1$ folgt, dass die Funktion $x+y$ auf $A$ konstant ist. Daher ist das Integral

$$\int_A (x+y) \, dA = \int_0^1 \int_0^{1-x} (x+y) \, dy \, dx.$$  

- Integrieren nach $y$ ergeben

$$\int_0^{1-x} (x+y) \, dy = \left[ xy + \frac{y^2}{2} \right]_0^{1-x} = x(1-x) + \frac{(1-x)^2}{2}.$$  

- Integrieren nach $x$ ergibt

$$\int_0^1 \left( x(1-x) + \frac{(1-x)^2}{2} \right) \, dx = \left[ \frac{x^2}{2} - \frac{x^3}{3} + \frac{(1-x)^3}{6} \right]_0^1 = \frac{1}{2} - \frac{1}{3} + \frac{1}{6} = \frac{1}{3}.$$  

**Antwort:** Das Integral $\int_A (x+y) \, dA$ ist $\frac{1}{3}$.

b) Seien $B = \{(x,y) \in \mathbb{R}^2 \mid 0 < x^2 + y^2 < 1, e^{x^2+y^2} \leq 1 \}$. Berechnen Sie das Integral

$$\int_B \sqrt{1 + e^{x^2+y^2}} \, dA.$$  

**Lösung:**

- Die Funktion $\sqrt{1 + e^{x^2+y^2}}$ ist über $B$ integrierbar.

- Integrieren nach $y$ ergibt

$$\int_0^{1-x^2} \sqrt{1 + e^{x^2+y^2}} \, dy = \left[ \sqrt{1 + e^{x^2+y^2}} \right]_0^{1-x^2} = \sqrt{1 + e^{x^2+1-x^2}} - \sqrt{1 + e^{x^2}}.$$  

- Integrieren nach $x$ ergibt

$$\int_0^1 \left( \sqrt{1 + e^{x^2+1-x^2}} - \sqrt{1 + e^{x^2}} \right) \, dx.$$  

**Antwort:** Das Integral $\int_B \sqrt{1 + e^{x^2+y^2}} \, dA$ ist abhängig von $B$.