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Boundary and Eigenvalue Problems

7. exercise sheet

Exercise 1

Consider the operator

$$Lu := -\partial_i \left(a^{ij}(x)\partial_j u\right) + 2b^i\partial_i u + (c + \partial_i b^i)u$$

whose coefficients a^{ij}, c, b_i are assumed to be bounded, measurable functions on a bounded domain $U \subset \mathbb{R}^n$. (We use the Einstein sum convention.) Furthermore we assume that b^i is $C^1(U)$ and c is non-negative. The coefficients a^{ij} satisfy a strict ellipticity conditions i.e. there exists $\lambda > 0$ such that

$$a^{ij}(x)\xi_i\xi_j \geq \lambda|\xi|^2 \text{ for all } x \in U \text{ and } \xi \in \mathbb{R}^n.$$

Show that $Lu = f$ can be solved for any $f \in W_0^{1,2}(U)^*$.

Exercise 2

Consider the bilinear form

$$a(u,v) := \int_{\mathbb{R}^n} Du(x) \cdot Dv(x) \text{ for } u,v \in H^1 := H^1(\mathbb{R}^n).$$

Let $A : H^1 \to H^1$ be the associated continuous operator i.e. $\langle Au, v\rangle = a(u,v)$ for all $u,v \in H^1$. $\langle \cdot,\cdot \rangle$ denotes the scalar product on H^1 (not the pairing with its dual).

1. Show that $\overline{A(H^1)} = H^1$.
2. Show that $A(H^1) \neq H^1$ i.e. the range of A is not closed.