Theorem 0.1 (Green’s Representation formula). Assume that $\Omega \subset \mathbb{R}^n$ is a $C^1$ domain and let $u \in C^2(\Omega)$. If $\gamma_n$ is the fundamental solution of the Laplace equation then for all $x \in \Omega$ we have that
\[
 u(x) = \int_{\Omega} \gamma_n(x - y)(-\Delta u(y))dy + \int_{\partial\Omega} \left( \gamma_n(x - y)\nabla u(y) - u(y)\nabla \gamma_n(x - y) \right) \cdot \nu(y)d\sigma_y,
\]
where $\nu(y)$ is the unit outward normal vector of $\partial\Omega$ at $y$.

Theorem 0.2 (Green’s function for the unit ball). The Green’s function for the ball $B_1(0)$ exists for $n \geq 2$ and it is given as follows (for $x \neq 0$ we define $x^* := \frac{x}{|x|^2}$):
\[
 G(x, y) = \begin{cases}
   \gamma_n(y - x) - \gamma_n(|x|x^* - y) & \text{if } x \neq 0 \\
   \gamma_n(y) - \frac{1}{(2-n)|\nu B_1(0)|} & \text{if } x = 0,
\end{cases}
\]

Theorem 0.3. Let $u \in C^2(\overline{B_1(0)})$ be a harmonic function with $u = f \in C(\partial B_1(0))$ on $\partial B_1(0)$. Then
\[
 u(x) = \frac{1}{|\partial B_1(0)|} \int_{\partial B_1(0)} \frac{f(y)}{|x - y|^n}d\sigma_y.
\]

Definition 0.4. Fundamental solution of $u_t(t, x) = \Delta_x u(t, x)$ in $(0, \infty) \times \mathbb{R}^n$: $\Phi(t, x) = \frac{1}{(4\pi t)^{\frac{n}{2}}} e^{-\frac{|x|^2}{4t}}$.

For all multiindices $\alpha$ and $b, d \in \mathbb{R}$ with $0 < b < d$ there exists $C_{\alpha, b}$ such that for all $t \in [b, d]$
\[
 |\partial_\alpha \Phi(t, x)| \leq C_{\alpha, b} e^{-\frac{|x|^2}{4t}}.
\]

Fourier series Let $f : \mathbb{R} \to \mathbb{C}$ be $T$-periodic function, $f|_{[-\pi, \pi]}$ integrierbar. Fourier coefficients
\[
 a_k = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \cos \left( \frac{2\pi k}{T} t \right) dt, \quad k \in \mathbb{N} \cup \{0\}, \quad b_k = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \sin \left( \frac{2\pi k}{T} t \right) dt, \quad k \in \mathbb{N},
\]

Fourier series of $f$: $\frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos \left( \frac{2\pi k}{T} t \right) + \sum_{k=1}^{\infty} b_k \sin \left( \frac{2\pi k}{T} t \right)$.

0.1 The wave equation in $\mathbb{R}$

We consider the initial value problem

\[
 \begin{cases}
   u_{tt} = c^2 u_{xx}, \quad (t, x) \in (0, \infty) \times \mathbb{R}, \\
   u(0, x) = g(x), \quad x \in \mathbb{R}, \\
   u_t(0, x) = h(x), \quad x \in \mathbb{R}.
\end{cases}
\]

Theorem 0.5. If $g \in C^2(\mathbb{R})$ and $h \in C^1(\mathbb{R})$, then the initial value problem (3) has a unique solution in $C^2([0, \infty) \times \mathbb{R})$, which is explicitly given by
\[
 u(t, x) = \frac{g(x + ct) + g(x - ct)}{2} + \frac{1}{2c} \int_{x-ct}^{x+ct} h(y) dy.
\]
A page from a document containing mathematical text is shown. The content is dense with mathematical expressions and theorems. The text includes an example, several theorems, and a corollary. It also references the Plancherel formula and uses notation such as $u_{tt} = u_{xx}$, $g$, $h$, $f$, $x$, $r$, $\tau$, $\beta$, $\xi$, $\Omega$, $\Delta$, and $\theta$. The page appears to be part of a book or article on partial differential equations or related topics.