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Classical methods for Partial Differential Equations

14. Problem Sheet

Exercise 1: (Damped string via Seperation of Variables again)
Let $f, g : \mathbb{R} \to \mathbb{R}$ be C^2, odd with $f(0) = f(\pi) = 0$ and $g(0) = g(\pi) = 0$. Moreover let f and g be 2π-periodic. Consider the damped wave equation

$$\begin{cases} u_{tt}(t,x) + \nu u_t(t,x) = c^2 u_{xx}(t,x)\,, & 0 < x < \pi\,,\, t > 0 \\ u(t,0) = u(t,\pi) = 0\,, \quad t > 0 & \\ u(0,x) = f(x)\,,\, u_t(0,x) = g(x) & 0 < x < \pi \end{cases}$$

and determine a general solution $u(t,x)$. Furthermore calculate a solution for $f(x) = sin(3x)$ and $g(x) = sin(5x)$ under the assumption that $c > \frac{\nu}{2}$.

Exercise 2: (Telegraph equation) Use properties of the Fourier transform to derive non-rigourosly a solution of the telegraph equation

$$\begin{cases} u_{tt}(t,x) + 2\nu u_t(t,x) = u_{xx}(t,x)\,, & t \in (0,\infty)\,,\, x \in \mathbb{R} \\ u(0,x) = f(x)\,,\, u_t(0,x) = g(x)\,, & x \in \mathbb{R} \end{cases}$$

where $f, g \in \mathcal{S}(\mathbb{R})$.

Exercise 3: (Non-compact identity mapping)
In Theorem 9.13 it is shown that for $U \in \mathbb{R}^n$ bounded and open, the identity map

$$i : W_0^{1,p}(U) \to L^p(U)\,, \qquad 1 \leq p < \infty$$

is compact.
What goes wrong if one tries to use the same proof in order to show that

$$i : W_0^{1,p}(\mathbb{R}^n) \to L^p(\mathbb{R}^n)\,, \qquad 1 \leq p < \infty$$

is compact.
Give a counterexample to shows that this identity map can not be compact.

Exercise 4:
Show the following assertions:

(i) $W_0^{k,p}(\mathbb{R}^n) = W^{k,p}(\mathbb{R}^n)$,

(ii) Let U be open and bounded and assume that ∂U has measure zero. Show that the only constant function in $W_0^{1,p}(U)$ is the zero function.