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Classical methods for Partial Differential Equations

03. Problem Sheet

Exercise 1:
Show that there exists up to a constant a unique spherical symmetric solution of the Poisson equation

$$-\Delta u = f \tag{1}$$

for arbitrary spherical symmetric $f \in C^2(\mathbb{R}^n)$ with compact support.

Exercise 2:

a) Consider a spherical symmetric function $f \in C^2(\mathbb{R}^2)$ that has compact support in an annulus with outer radius R and inner radius r with $0 < r < R$. Show that the unique spherical symmetric solution u of the Poisson equation (1) is constant in $B_r(0)$.

b) Consider a ball with radius $R > 0$ centered around the origin in $\mathbb{R}^3$. Moreover suppose the mass density ρ of this ball is spherical symmetric and two times continuously differentiable. Calculate the gravitational field of the ball for all $x \in \mathbb{R}^3$.

Exercise 3:
Let $A := \{(x_1, \dots, x_n) \in \mathbb{R}^n \ : \ x_{n-1} > 0 \text{ and } x_n > 0\}$ be the first quadrant. Determine the Green's function on the space A.

Exercise 4:
Let A be as in Exercise 3 and $f \in C_c^2(A)$. Moreover consider the differential equation

$$\begin{cases} -\Delta u = f & \text{in } A \\ u = 0 & \text{on } \partial A \end{cases}$$

Show that **a** solution u is given by

$$u(x) = \int G(x,y) f(y) dy\,.$$