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Classical methods for Partial Differential Equations

08. Problem Sheet

Exercise 1: (Damped string)
Let $f : \mathbb{R} \to \mathbb{R}\,;\, x \mapsto \sin(x)$ and $g : \mathbb{R} \to \mathbb{R}\,;\, x \mapsto \cos(x)$. Consider the damped wave equation

$$\begin{cases} u_{tt} + \nu u_t = c^2 u_{xx}\,, & 0 < x < \pi\,,\ t > 0 \\ u(t,0) = u(t,\pi) = 0\,, & t > 0 \\ u(0,x) = f(x)\,,\ u_t(0,x) = g(x) & 0 < x < \pi \end{cases}$$

Determine a solution $u(t,x)$.

Exercise 2: (Clamped plucked string)
Calculate via seperation of variable a solution of the following problem

$$\begin{aligned} u_{tt} - c^2 u_{xx} &= 0 && x \in [0,L], t \in [0,\infty), \\ u(x,0) &= u_0(x), && x \in [0,L], \\ u_t(x,0) &= u_1(x), && x \in [0,L], \\ u(t,0) = u(t,L) &= 0, && t \geq 0, \end{aligned}$$

where

$$u_0(x) = \begin{cases} \frac{Hx}{\alpha L} & ,0 \leq x \leq \alpha L \\ \frac{H}{\alpha - 1}\left(\frac{x}{L} - 1\right) & ,\alpha L \leq x \leq L \end{cases} \qquad \text{and} \qquad u_1(x) = 0,$$

Note that $H, L > 0$ and $0 < \alpha < 1$ are given constants.

Exercise 3:
Calculate a solution of the following problem:

$$\begin{aligned} \Delta u(x,y) &= 0, && 1 < \sqrt{x^2+y^2} < 3, \\ u(x,y) &= 3, && \sqrt{x^2+y^2} = 1, \\ u(x,y) &= 3 + x, && \sqrt{x^2+y^2} = 3. \end{aligned}$$

Hint: First transform the problem into polar coordinates (r, φ) and then use seperation of variables.