

Funktionentheorie Exercise Sheet 1

Exercise 1(C) (10 points)

- (a) Find the real and the imaginary parts of the following numbers

(1) $(2 + i)^3$; (2) $\frac{3+4i}{1+(1-i)^2}$
and write them in polar coordinates.

- (b) Solve the following equations

(1) $z^3 - 3z^2 + 6z - 4 = 0$;
(2) $z^2 - 2\bar{z} + 1 = 0$;
(3) $z^3 = \frac{2+10i}{2-3i}$.

Exercise 2(C) (10 points)

- (a) A complex number z is called an n -th root of unity ($n \in \mathbb{N}$) if $z^n = 1$. From the lecture we know that for each $n \in \mathbb{N}$ there are exactly n different n -th roots of unity, given by

$$z_k = \cos \frac{2k\pi}{n} + i \sin \frac{2k\pi}{n}, \quad 0 \leq k \leq n-1.$$

Sketch the n -th roots of unity for $n = 1, \dots, 5$. Conjecture how the n -th roots of unity look like for $n \geq 6$.

- (b) Sketch the following subsets of \mathbb{C}

(1) $\{z \in \mathbb{C} : 1 < \operatorname{Re}(iz) < 2\}$;
(2) $\{z \in \mathbb{C} : \operatorname{Re}(z^2) < 1\}$;
(3) $\{z \in \mathbb{C} : |z - i| > 1 \text{ and } |z - 2i + 1| \leq 4\}$;
(4) $\{z \in \mathbb{C} : |z - 2i + 1| = |z + 3i - 2|\}$.

Exercise 3 Prove that for $n \in \mathbb{N}$ it holds

$$\prod_{k=1}^{n-1} \sin \frac{k\pi}{n} = \frac{n}{2^{n-1}}.$$

Hint: You may use that $z^n - 1 = \prod_{k=1}^n (z - \zeta^k)$ with $\zeta := \cos\left(\frac{2\pi}{n}\right) + i \sin\left(\frac{2\pi}{n}\right)$.

Exercise 4 Let $z_0 = x_0 + iy_0 \neq 0$ be a given complex number. Define the sequence $\{z_n\}_{n \geq 0}$ recursively by

$$z_{n+1} = \frac{1}{2} \left(z_n + \frac{1}{z_n} \right).$$

Show that

- (a) if $x_0 > 0$ then $\lim_{n \rightarrow \infty} z_n = 1$;
(b) if $x_0 < 0$ then $\lim_{n \rightarrow \infty} z_n = -1$;
(c) if $x_0 = 0, y_0 \neq 0$ then $\{z_n\}_{n \geq 0}$ is undefined or divergent.

Hint: Consider $w_{n+1} = \frac{z_{n+1}-1}{z_{n+1}+1}$.

Important Information

- Information about the lecture can be found at

<http://www.math.kit.edu/iana1/lehre/ft2012s/>

- Every Tuesday a new Exercise Sheet comes out. It can be downloaded from the above mentioned webpage. On each Exercise Sheet there are two exercises marked with **(C)**. These may be submitted for correction. Your solutions must be handed until 13:30 on the next Tuesday (e.g. Exercises 1 & 2 from Exercise Sheet 1 must be handed until 13:30 on Tuesday, April 24) into the **Funktionentheorie**-box next to Room 3A-03 in the Allianz-Gebäude. The corrected solutions can be found in the **Funktionentheorie**-box next to Room 3A-15 in the Allianz-Gebäude. Please write each Exercise on a separate sheet of paper with your name and matriculation number.
- Students who need Exercise Certificate (Übungsschein) must acquire 50% of the points from the first 7 Exercise Sheets plus 50% of the points from the second 7 Exercise Sheets.
- The lecture scheduled for 28.05.2012 will be replaced by a lecture on 01.06.2012 (Friday).
The exercise class scheduled for 01.05.2012 will be replaced by an exercise class on 04.05.2012 (Friday).
In the last week an additional exercise class will take place on 20.07.2012 (Friday).
All the rescheduled lecture/exercise classes will take place in Kl HS (Geb. 10.50) at 11:30h.
- The examination in Funktionentheorie will take place on 24.07.2012 between 10h and 12h.