**Exercise 7: Closed Subspaces**

Show that the following subsets form closed linear subspaces of given normed spaces:

1. Consider a complete normed space \((B(X), \| \cdot \|_\infty)\) of bounded functions on a metric space \((X, d)\) with supremum norm and following sets:
   
   (a) bounded continuous functions \(C_b(X) = \{ f : X \to \mathbb{K} \mid f \text{ is bounded and continuous} \}\),
   
   (b) bounded uniformly continuous functions \(BUC(X) = \{ f : X \to \mathbb{K} \mid f \text{ is bounded and uniformly continuous} \}\).

2. Consider a complete normed space \((l^\infty, \| \cdot \|_\infty)\) of bounded sequences in \(\mathbb{K}\) with supremum norm and set of converging sequences \(c = \{ x = (x_j)_{j \in \mathbb{N}} \in \mathbb{K}^\mathbb{N} \mid \lim_{j \to \infty} x_j \text{ exists in } \mathbb{K} \}\).

**Exercise 8: Norm of Bounded Operator**

Consider a linear operator defined on a normed space \((\mathbb{K}^n, \| \cdot \|_p)\) as \(T : x \mapsto Ax\) where \(A = (a_{jk})_{j,k=1}^n\) is a matrix. Calculate norm of \(T\) for

1. \(p = 1\),
2. \(p = \infty\).

**Exercise 9: Integral Operator**

Consider a linear operator defined on a normed space \((C([0, 1]), \| \cdot \|_\infty)\) as \(T : f \mapsto Tf(t) := \int_0^t f(\tau) d\tau\). Show the following properties

1. \(\|Tf\|_\infty \leq \|f\|_\infty\),
2. \(\|T\|_\infty = 1\),
3. \(T^n f(t) = \int_0^t \frac{(t-\tau)^{n-1}}{(n-1)!} f(\tau) d\tau\),
4. \(\|T^n f\|_\infty \leq \frac{\|f\|_\infty}{n!}\),
5. \(\|T^n\|_\infty = \frac{1}{n!}\).

**Remark:** The exercises marked with a star can be handed in for the correction till Wednesday noon to the letterbox at the ground level of the Math building 20.30.

http://www.math.kit.edu/iana1/edu/funcana2018w/en