**Exercise 32:**
Let \(X, Y, Z\) be Banach spaces where \(Y, Z\) are linear subspaces of a vector space \(W\). Assume that \(d\) is a metric on \(W\) such that identities

\[
I_Y : (Y, \|\cdot\|_Y) \to (W, d), y \mapsto y \quad \text{and} \quad I_Z : (Z, \|\cdot\|_Z) \to (W, d), z \mapsto z
\]

are continuous. Let \(T \in \mathcal{L}(X, Y)\) and suppose \(T(X) \subseteq Z\). Show that \(T \in \mathcal{L}(X, Z)\).

**Exercise 33:**
Let \(X, Y\) be Banach spaces and let \(T : D(T) \to Y\) be a linear operator with \(D(T) \subseteq X\). Show the following

1. \(T\) is closed \(\iff\) \(D(T)\) is a Banach space for the graph norm \(\|x\|_T := \|x\|_X + \|Tx\|_Y\) for \(x \in D(T)\).
2. \(T \in \mathcal{L}((D(T), \|\cdot\|_T), Y)\).

**Exercise 34:**
Let \(X\) be Banach space and \(U \subseteq X\) be a closed linear subspace. Find canonical isomorphisms which shows that

\[
(X/U)' \simeq U^\perp \quad \text{and} \quad U' \simeq X'/U^\perp.
\]

**Exercise 35:**
Show the following

1. Every \(\mathbb{K}\)-vector space has a basis.
2. If \(X\) is a normed space and \(\dim X = \infty\) then there exist \(\varphi : X \to \mathbb{K}\) which are linear and unbounded.

**Remark:** The exercises marked with a star can be handed in for the correction till Wednesday noon to the letterbox at the ground level of the Math building 20.30.

http://www.math.kit.edu/iana1/edu/funcana2018w/en