Erin 1: $z^n = r e^{i\varphi}$ hat die Lösungen $z_k = r^{\frac{1}{n}} e^{i\frac{\varphi + 2\pi k}{n}}$, $k = 0, 1, \dots, n-1$

Erin 2: Ist $w = a + bi$, $a, b \in \mathbb{R}$.

dann: $w = |w| e^{i\varphi}$ wobei

$$\varphi = \begin{cases} \arccos\left(\frac{a}{\sqrt{a^2+b^2}}\right), & \text{wenn } b \geq 0 \\ -\arccos\left(\frac{a}{\sqrt{a^2+b^2}}\right), & \text{wenn } b \leq 0. \end{cases}$$

Bestimmen alle Lösungen der Gleichung $z^5 = -\frac{1+3i}{2+i}$.

$$-\frac{1+3i}{2+i} = -\frac{(1+3i)(2-i)}{(2+i)(2-i)} = -\frac{2-i+6i-3i^2}{2^2-i^2}$$

$$= -\frac{5+5i}{5} = -1-i.$$

$|-1-i| = \sqrt{1^2+1^2} = \sqrt{2}$. Also

und da $\text{Im}(-1-i) = -1 < 0$.

folgt $-1-i = \sqrt{2}\, e^{i\theta}$

wobei $\theta = -\arccos\left(\frac{\text{Re}(-1-i)}{\sqrt{2}}\right) = \arccos\left(-\frac{1}{\sqrt{2}}\right)$.

$$\Rightarrow \quad \theta = -\frac{3\pi}{4}.$$

Also $z^5 = \sqrt{2}\, e^{-i\theta\frac{3\pi}{4}}$.

$$\Rightarrow \quad z_k = (\sqrt{2})^{\frac{1}{5}}\, e^{\frac{i\theta}{5}\left(-\frac{3\pi}{4}+2k\pi\right)} \quad k = 0,1,2,3,4.$$