

3.13. Logarithmus [Sei  $z \in \mathbb{C} \setminus \{0\}$ . Dann <sup>(73)</sup>

$z = |z| e^{i\varphi}$ ,  $\varphi$  heißt Argument von  $z$   
( $\arg z$ ).  $\varphi + 2k\pi$  ist auch ein Argument  
von  $z$ , wenn  $k \in \mathbb{Z}$ .

Sei  $z \in \mathbb{C} \setminus (-\infty, 0]$ . Dann gibt es  
 $\theta \in (-\pi, \pi)$  eindeutig mit  $z = |z| e^{i\theta}$ .  
 $\theta$  heißt Hauptzweig des Arguments von  $z$ .  
( $\theta = \text{Arg } z$ ). Wir definieren dann

(\*)  $\text{Log } z = \ln |z| + i \text{Arg } z$ ,  $z \in \mathbb{C} \setminus (-\infty, 0]$   
(Hauptzweig des Logarithmus)

Bsp  $z = 2i$ . Dann  $z = 2e^{i\frac{\pi}{2}}$ . Also

$\text{Arg } z = \frac{\pi}{2}$ , da  $\frac{\pi}{2} \in (-\pi, \pi)$ . Also

$$\text{Log } 2i \stackrel{(*)}{=} \ln |2i| + i \frac{\pi}{2} = \ln 2 + i \frac{\pi}{2}. \quad (1)$$

Bem 3.13.2:  $\text{Log}: \mathbb{C} \setminus (-\infty, 0] \rightarrow \{w \in \mathbb{C} : |\text{Im } w| < \pi\}$   
ist holomorph und bijektiv mit

Umkehrabbildung exp. Es gilt

(74)

$$\text{Log}' z = \frac{1}{z}, \quad z \in \mathbb{C} \setminus (-\infty, 0].$$

3.13.3. Allgemeine Potenz: Für  $z \in \mathbb{C} \setminus (-\infty, 0]$  und  $\alpha \in \mathbb{C}$  definiert man durch

$$z^\alpha := \exp(\alpha \text{Log} z)$$

den Hauptzweig der  $\alpha$ -ten Potenz.

$$\begin{aligned} \text{Bsp } (2i)^i &= \exp(i \text{Log}(2i)) \stackrel{(1)}{=} \\ &= \exp(i (\ln 2 + i \frac{\pi}{2})) = \exp(-\frac{\pi}{2} + i \ln 2) \\ &= e^{-\frac{\pi}{2}} e^{i \ln 2} \end{aligned}$$

3.13.4 Für  $n \in \mathbb{N}$  enthält man den Hauptzweig der  $n$ -ten Wurzel

$$z^{\frac{1}{n}} = |z|^{\frac{1}{n}} \exp(i \text{Arg} z / n).$$

Bsp

$$\begin{aligned} (2i)^{\frac{1}{3}} &= |2i|^{\frac{1}{3}} \exp(i (\frac{\pi}{2}) \frac{1}{3}) = 2^{\frac{1}{3}} \exp(i \frac{\pi}{6}) \\ &= 2^{\frac{1}{3}} \left( \cos\left(\frac{\pi}{6}\right) + i \sin\left(\frac{\pi}{6}\right) \right) = 2^{\frac{1}{3}} \left( \frac{\sqrt{3}}{2} + i \frac{1}{2} \right). \end{aligned}$$

# Fouriertransformation

(75)

Erinnerung: Ist  $f$   $2\pi$ -periodisch und stetig dann

$$f(t) = \sum_{n=-\infty}^{\infty} a_n e^{int} \quad \forall t \in \mathbb{R} \quad (1), \quad \text{wobei} \quad a_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) e^{-int} dt, \quad (2)$$

**Beitrag / Intensität  
der Schwingung  $e^{int}$**

Herleitung von (2) (nicht rigoros)

$$\begin{aligned} \int_{-\pi}^{\pi} f(t) e^{-int} dt &\stackrel{(1)}{=} \int_{-\pi}^{\pi} \sum_{m=-\infty}^{\infty} a_m e^{imt} e^{-int} dt \\ &= \sum_{m=-\infty}^{\infty} a_m \underbrace{\int_{-\pi}^{\pi} e^{i(m-n)t} dt}_{= \begin{cases} 2\pi, & \text{wenn } m=n \\ 0, & \text{wenn } m \neq n \end{cases}} = 2\pi a_n \Rightarrow a_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) e^{-int} dt \end{aligned}$$

Informale Herleitung

Ähnlich ist  $f$   $T$ -periodisch und stetig,

dann

$$f(t) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \underbrace{b_{\frac{n2\pi}{T}} \frac{2\pi}{T}}_{\text{"unendliche Riemansche Summe"}} e^{+i \frac{n2\pi}{T} t}$$

"unendliche Riemansche Summe"

wobei

$$b_{\frac{n2\pi}{T}} = \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-i \frac{n2\pi}{T} t} dt$$

Was passiert wenn  $f$  nicht 76  
periodisch ist? Wir betrachten

den Limes  $T \rightarrow \infty$ . Dann alle  
Schwingungen  $e^{i\omega t}$  haben Beitrag,

die "Riemannsche Summe" wird Integral und

mit bekommen,  $b_\omega = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$ .

und  $f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} b_\omega e^{i\omega t} d\omega$ .

Diese informale Herleitung führt zur  
folgenden Definition:

Def 4.1. Sei  $f: \mathbb{R} \rightarrow \mathbb{C}$  stückweise  
stetig und absolut integrierbar (aib), d.h.  
 $\int_{-\infty}^{\infty} |f(t)| dt < \infty$ . Die Fouriertransformierte  
von  $f$  definiert man für  $\omega \in \mathbb{R}$  durch

$$\tilde{f}(\omega) := \int_{-\infty}^{\infty} e^{-i\omega t} f(t) dt.$$

[4.2 Fourierinversionsformel. Sei  $f: \mathbb{R} \rightarrow \mathbb{C}$  (77)

wie in Def 4.1, ist  $f$  auch (aib),  
dann

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{it\omega} \tilde{f}(\omega) d\omega \quad \forall t \in \mathbb{R}$$

Interpretation:  $f$  ist kontinuierliche Summe  
von Schwingungen  $e^{it\omega}$  und  $\frac{\tilde{f}(\omega)}{2\pi}$  entspricht  
den Beitrag der Schwingung  $e^{it\omega}$ .

Bsp  $f(t) = e^{-a|t|}$ ,  $a > 0$ .

Dann  $\tilde{f}(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t} e^{-a|t|} dt$ .

$$= \int_{-\infty}^0 e^{-i\omega t} e^{at} dt + \int_0^{\infty} e^{-i\omega t} e^{-at} dt \quad (1)$$

$$\int_0^{\infty} e^{-i\omega t} e^{-at} dt = \lim_{b \rightarrow \infty} \int_0^b e^{-(a+i\omega)t} dt$$

$$= \lim_{b \rightarrow \infty} \left( \frac{e^{-(a+i\omega)t}}{a+i\omega} \Big|_0^b \right) = \lim_{b \rightarrow \infty} \frac{1 - e^{-(a+i\omega)b}}{a+i\omega} = \frac{1}{a+i\omega} \quad (2)$$

ähnlich  $\int_{-\infty}^0 e^{-i\omega t} e^{at} dt = \frac{1}{a-i\omega} \quad (3)$

Aus (1), (2), (3) folgt.

$$\mathcal{F} f(\omega) = \frac{1}{a+i\omega} + \frac{1}{a-i\omega} = \frac{a-i\omega + a+i\omega}{a^2 - (i\omega)^2} \quad (78)$$

$$\Rightarrow \mathcal{F} f(\omega) = \frac{2a}{a^2 + \omega^2}$$

### 4.3 Rechenregeln

Die Fouriertransformation hat aufgrund ihrer Ähnlichkeit mit der Laplace-Transformation ähnliche Eigenschaften wie die Laplace-Transformation, und zwar:

[ Ist  $f: \mathbb{R} \rightarrow \mathbb{C}$  stückweise stetig und (aib) dann

(a)  $\forall a \in \mathbb{R} \setminus \{0\}$  gilt  $\mathcal{F}\{f(at)\}(\omega) = \frac{1}{|a|} \mathcal{F}f\left(\frac{\omega}{a}\right), \omega \in \mathbb{R}$

(b)  $\forall b \in \mathbb{R}$  gilt  $\mathcal{F}\{f(t-b)\}(\omega) = e^{-i\omega b} \mathcal{F}f(\omega), \omega \in \mathbb{R}$

und  $\mathcal{F}\{e^{i\omega b} f(t)\}(\omega) = \mathcal{F}f(\omega - b)$

(c) Ist  $f$  stetig und stückweise differenzierbar und  $f'$  wieder stückweise stetig und (aib) dann  $\mathcal{F}\{f'\}(\omega) = i\omega \mathcal{F}f(\omega)$ .

(d) Ist  $t \mapsto t^n f(t)$  ( $n \in \mathbb{N}$ ) absolut integrierbar, dann ist  $\mathcal{F}f$   $n$ -Mal differenzierbar und

$$(\mathcal{F} f)^{(n)}(\omega) = \mathcal{F} \{ (-it)^n f(t) \}(\omega), \omega \in \mathbb{R} \quad (79)$$

Manche Beweisideen: (a) Für  $a > 0$

$$\mathcal{F} \{ f(at) \}(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t} f(at) dt$$

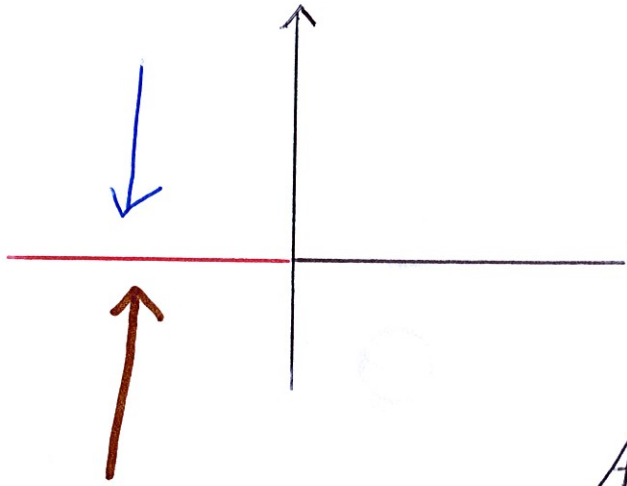
$$\frac{s=at}{t=\frac{s}{a}, dt=\frac{ds}{a}} \quad \int_{-\infty}^{\infty} e^{-i\omega \frac{s}{a}} f(s) \frac{ds}{a}$$

$$= \frac{1}{a} \underbrace{\int_{-\infty}^{\infty} e^{-i\frac{\omega}{a}s} f(s) ds}_{\mathcal{F} f\left(\frac{\omega}{a}\right)} = \frac{1}{a} \mathcal{F} f\left(\frac{\omega}{a}\right)$$

Bemerkung: Wenn man ~~(a)~~ ~~(b)~~ ~~(c)~~ ~~(d)~~ ~~(e)~~ ~~(f)~~ ~~(g)~~ ~~(h)~~ ~~(i)~~ ~~(j)~~ ~~(k)~~ ~~(l)~~ ~~(m)~~ ~~(n)~~ ~~(o)~~ ~~(p)~~ ~~(q)~~ ~~(r)~~ ~~(s)~~ ~~(t)~~ ~~(u)~~ ~~(v)~~ ~~(w)~~ ~~(x)~~ ~~(y)~~ ~~(z)~~ ~~(aa)~~ ~~(ab)~~ ~~(ac)~~ ~~(ad)~~ ~~(ae)~~ ~~(af)~~ ~~(ag)~~ ~~(ah)~~ ~~(ai)~~ ~~(aj)~~ ~~(ak)~~ ~~(al)~~ ~~(am)~~ ~~(an)~~ ~~(ao)~~ ~~(ap)~~ ~~(aq)~~ ~~(ar)~~ ~~(as)~~ ~~(at)~~ ~~(au)~~ ~~(av)~~ ~~(aw)~~ ~~(ax)~~ ~~(ay)~~ ~~(az)~~ ~~(ba)~~ ~~(bb)~~ ~~(bc)~~ ~~(bd)~~ ~~(be)~~ ~~(bf)~~ ~~(bg)~~ ~~(bh)~~ ~~(bi)~~ ~~(bj)~~ ~~(bk)~~ ~~(bl)~~ ~~(bm)~~ ~~(bn)~~ ~~(bo)~~ ~~(bp)~~ ~~(bq)~~ ~~(br)~~ ~~(bs)~~ ~~(bt)~~ ~~(bu)~~ ~~(bv)~~ ~~(bw)~~ ~~(bx)~~ ~~(by)~~ ~~(bz)~~ ~~(ca)~~ ~~(cb)~~ ~~(cc)~~ ~~(cd)~~ ~~(ce)~~ ~~(cf)~~ ~~(cg)~~ ~~(ch)~~ ~~(ci)~~ ~~(cj)~~ ~~(ck)~~ ~~(cl)~~ ~~(cm)~~ ~~(cn)~~ ~~(co)~~ ~~(cp)~~ ~~(cq)~~ ~~(cr)~~ ~~(cs)~~ ~~(ct)~~ ~~(cu)~~ ~~(cv)~~ ~~(cw)~~ ~~(cx)~~ ~~(cy)~~ ~~(cz)~~ ~~(da)~~ ~~(db)~~ ~~(dc)~~ ~~(dd)~~ ~~(de)~~ ~~(df)~~ ~~(dg)~~ 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(80)

Bemerkung: Eine Erklärung folgt, warum  $\operatorname{Log} z$  nur in  $\mathbb{C} \setminus (-\infty, 0]$  definiert ist.



Grund: Wenn man  $(-\infty, 0]$  von oben approximiert (siehe Skizze) dann

$\operatorname{Arg} z \rightarrow \pi$ . Wenn man

aber  $(-\infty, 0]$  von unten

approximiert, dann  $\operatorname{Arg} z \rightarrow -\pi$ .

Deswegen wenn man versucht  $\operatorname{Log} z$  auf  $(-\infty, 0]$  fortzusetzen, bekommt man eine Funktion, die nicht stetig ist.

(da  $\operatorname{Log} z = \ln|z| + i \operatorname{Arg} z$ ).