Problems

1. Let a self-adjoint operator $A$ be bounded from below by $\gamma$. Assume that $q$ is relatively form bounded by a form associated to $A$ with constants $a$ and $b$. Show that the operator $C_q(\lambda)$ associated with a quadratic form $q((A + \lambda)^{-1/2}\psi)$ satisfies $\|C_q(\lambda)\| \leq \max\{a, \frac{b}{\lambda + \gamma}\}$ for $\lambda > -\gamma$.

2. Let a self-adjoint operator $A$ be bounded from below by $\gamma$. Assume that $C_q(\lambda)$ is a bounded operator for sufficiently large $\lambda$. Show the following:
   (a) for $a := a(\lambda) = \|C_q(\lambda)\|$ there exists $b \geq 0$ such that
   \[
   |q(\varphi)| \leq a q_{A-\gamma}(\varphi) + b \|\varphi\|^2
   \]
   holds $\forall \varphi \in Q(A)$,
   (b) $\lim_{\lambda \to \infty} \|C_q(\lambda)\| = a_0$ where $a_0 = \inf\{a \in \mathbb{R}^+ \mid (1) \text{ holds for } a \text{ and some } b \geq 0\}$.