Mathematical Methods of Quantum Mechanics
Exercise 12

Problems

1. Let a self-adjoint operator A be bounded from below by γ. Assume that q is relatively form bounded by a form associated to A with constants a and b. Show that the operator $C_q(\lambda)$ associated with a quadratic form $q((A+\lambda)^{-1/2}\psi)$ satisfies $\|C_q(\lambda)\| \leq \max\left\{a, \frac{b}{\lambda+\gamma}\right\}$ for $\lambda > -\gamma$.
2. Let a self-adjoint operator A be bounded from below by γ. Assume that $C_q(\lambda)$ is a bounded operator for sufficiently large λ. Show the following:
   (a) for $a := a(\lambda) = \|C_q(\lambda)\|$ there exists $b \geq 0$ such that
   $$|q(\varphi)| \leq a q_{A-\gamma}(\varphi) + b\|\varphi\|^2 \tag{1}$$
   holds $\forall \varphi \in \mathcal{Q}(A)$,
   (b) $\lim_{\lambda\to\infty} \|C_q(\lambda)\| = a_0$ where $a_0 = \inf\{a \in \mathbb{R}^+ | (1) \text{ holds for } a \text{ and some } b \geq 0\}$.