Mathematical Methods of Quantum Mechanics
Exercise 13

Problems

1. **Newton's Shell Theorem** Prove that the following equality holds

$$\int_{|x|=r} \frac{d\sigma(x)}{|x-R|} = \frac{4\pi r^2}{\max\{r,|R|\}}$$

   where $d\sigma(x)$ is a surface measure on a sphere and $R \in \mathbb{R}^3$.

2. Let $H = -\Delta + V$ be Schrödinger operator on $H^2(\mathbb{R}^3)$ with $V \in L^2(\mathbb{R}^3) + L^\infty(\mathbb{R}^3)$. Furthermore assume that $\lim_{|x|\to\infty} V(x) = a$. Show that the essential spectrum is $\sigma_{ess}(H) = [a, \infty)$.
   Remark: For $[a,\infty) \subset \sigma_{ess}(H)$ construct appropriate test functions.
   For $\inf \sigma_{ess}(H) \geq a$ try to use IMS formula.