Problems

1. **Newton’s Shell Theorem** Prove that the following equality holds

   \[
   \int_{|x|=r} \frac{d\sigma(x)}{|x-R|} = \frac{4\pi r^2}{\max\{r, |R|\}}
   \]

   where \(d\sigma(x)\) is a surface measure on a sphere and \(R \in \mathbb{R}^3\).

2. Let \(H = -\Delta + V\) be Schrödinger operator on \(H^2(\mathbb{R}^3)\) with \(V \in L^2(\mathbb{R}^3) + L^\infty(\mathbb{R}^3)\). Furthermore assume that \(\lim_{|x| \to \infty} V(x) = a\). Show that the essential spectrum is \(\sigma_{ess}(H) = [a, \infty)\).

   **Remark:** For \([a, \infty) \subset \sigma_{ess}(H)\) construct appropriate test functions.

   For \(\inf \sigma_{ess}(H) \geq a\) try to use IMS formula.