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# Mathematical Methods in Quantum Mechanics II

## 12th Exercise Sheet

**Exercise 36:**
Using Newton's theorem show that

$$\left\langle \phi\otimes\phi, \frac{1}{|z_1 - z_2 - y|}\ \phi\otimes\phi \right\rangle = \left\langle \phi\otimes\phi, \frac{1}{|z_2 + y|}\ \phi\otimes\phi \right\rangle$$

for all $y \in \mathbb{R}^3 \setminus \{0\}$ and radially symmetric $\phi \in L^2(\mathbb{R}^3)$ supported in the ball $B(0, \frac{|y|}{3})$.

**Exercise 37:**
For given $y \in \mathbb{R}^3 \setminus \{0\}$ show that there are functions $C^\infty(\mathbb{R}^3) \ni j_1, j_2, j_3 : \mathbb{R}^3 \to [0,1]$ such that $j_1^2 + j_2^2 + j_3^2 = 1$ and

$$j_1(x) = 1 \text{ if } |x| \leq \frac{1}{3} \text{ and } j_1(x) = 0 \text{ if } |x| \geq \frac{1}{2}$$

and

$$j_2(x) = 1 \text{ if } |x - \frac{y}{|y|}| \leq \frac{1}{3} \text{ and } j_2(x) = 0 \text{ if } |x - \frac{y}{|y|}| \geq \frac{1}{2}.$$

**Exercise 38:**
For the Hamiltonian

$$H(y) = -\Delta_{x_1} - \frac{1}{|x_1|} - \Delta_{x_2} - \frac{1}{|x_2 - y|} + \frac{1}{|y|} + \frac{1}{|x_1 - x_2|} - \frac{1}{|x_1 - y|} - \frac{1}{|x_2|}$$

let $E(y) = \inf \sigma(H(y))$. Following the proofs of HVZ and Zishlin's theorem from last semester show that $E(y) \in \sigma_{\text{disc}}(H(y))$.