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Mathematical Methods in Quantum Mechanics II

13th Exercise Sheet

Exercise 39:
Show that for $y, z \in \mathbb{R}^3$ and $|z| \leq \frac{2|y|}{3}$ we have the expansion

$$\frac{1}{|z+y|} = \frac{1}{|y|} - \frac{z \cdot \hat{y}}{|y|^2} + \frac{3(z \cdot \hat{y})^2 - |z|^2}{2|y|^3} + O\Big(\frac{|z|^3}{|y|^4}\Big)$$

where $\hat{y} = \frac{y}{|y|}$.

Exercise 40:
Following the notation given in class (on the 17/07/2020 lecture) show

$$\Big\langle P^{\perp} H\phi \otimes \phi_y, (\tilde{H}^{\perp} - E(\infty))^{-1} P^{\perp} H\phi \otimes \phi_y \Big\rangle =$$

$$\Big\langle I\phi \otimes \phi, (H_0^{\perp} - E(\infty))^{-1} I\phi \otimes \phi \Big\rangle + O(e^{-c|y|})$$

where

$$H_0 = -\Delta_{z_1} - \frac{1}{|z_1|} - \Delta_{z_2} - \frac{1}{|z_2|}$$

and c is a positive constant.

Exercise 41:
For $y \in \mathbb{R}^3 \setminus \{0\}$ and $c > 0$ let

$$\delta = \delta(x_1, x_2) = c\Big((1 + |x_1|^2)^{\frac{1}{2}} + (1 + |x_2 - y|^2)^{\frac{1}{2}}\Big).$$

Show that the operator

$$e^{\delta}(-\Delta + 1)^{-1} e^{-\delta}$$

is bounded from $L^2(\mathbb{R}^6) \to L^2(\mathbb{R}^6)$ for sufficiently small c.