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Mathematical Methods in Quantum Mechanics II

7th Exercise Sheet

Exercise 20:
Prove that the operators $a(f), a^*(f)$, where $f \in L^2(\mathbb{R}^3)$, are unbounded on $\mathcal{F}_s(L^2(\mathbb{R}^3))$ but bounded on $\mathcal{F}_a(L^2(\mathbb{R}^3))$.

Exercise 21:
Show that a semi-bounded quadratic form $q : Q(q) \times Q(q) \to \mathbb{C}$ is closed if and only if for every sequence $\{\phi_n\}_n \subset Q(q)$ and $\phi \in \mathcal{H}$ with $\phi_n \to \phi$ in $\mathcal{H}$ and $q(\phi_n - \phi_m, \phi_n - \phi_m) \to 0$ as $n, m \to \infty$ we have that $\phi \in Q(q)$ and $q(\phi_n - \phi, \phi_n - \phi) \to 0$ as $n \to \infty$.

Using the above criterion show that the quadratic form $q : C_c(\mathbb{R}) \times C_c(\mathbb{R}) \to \mathbb{C}$ in $L^2(\mathbb{R})$ given by $q(f, g) = \overline{f(0)}g(0)$ is not closed. Moreover, prove that q does not have any closed extensions.

Exercise 22:
Show that the quadratic form $q(\phi, \psi) + \frac{5}{4}\langle \phi, \psi \rangle$ where $q : H^1(\mathbb{R}^3) \times H^1(\mathbb{R}^3) \to \mathbb{C}$ is defined as

$$q(\phi, \psi) = \int_{\mathbb{R}^3} \left[\overline{\nabla \phi(x)}\ \nabla \psi(x) - \frac{\overline{\phi(x)}\psi(x)}{|x|} \right] dx$$

is associated with the operator $-\Delta - \frac{1}{|x|} + \frac{5}{4}$.