

Mathematical Quantum Mechanics 2
(aka. Mathematical Methods in Quantum Mechanics 2)
3. Homework

Exercise 5

Consider the operator

$$H = -\Delta + V$$

on $L^2(\mathbb{R}^3)$ for some potential $V \in C(\mathbb{R}^3)$ such that $\lim_{|x| \rightarrow \infty} V(x) = \infty$.

- a) Show that H has solely discrete spectrum.
- b) Let ψ an eigenfunction of H , i.e. there exists an $E \in \mathbb{R}$ with $H\psi = E\psi$. Show

$$\exp(\alpha|\cdot|)\psi \in L^2(\mathbb{R}^3) \quad \forall \alpha \in \mathbb{R}.$$

Exercise 6

Consider the operator

$$H = -\Delta + V$$

on $L^2(\mathbb{R}^3)$ for some $V \in C(\mathbb{R}^3)$ such that there exist $c, R > 0$ with $V(x) \geq \frac{c}{|x|}$ for any $|x| \geq R$. Let $F_\mu(x) := 2\mu|x|^{1/2}$. Show for any eigenfunction ψ of H that

$$\exp[F_\mu]\psi \in L^2(\mathbb{R}^3)$$

for any $\mu < c^2$.