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Mathematical Quantum Mechanics 2
(aka. Mathematical Methods in Quantum Mechanics 2)
6. Homework

Exercise 14

Let $\gamma \in \mathcal{S}_1(\mathcal{H})$ be self-adjoint and positive with $\operatorname{tr}(\gamma) = 1$ and let $\varrho = |\varphi\rangle\langle\varphi|$ for some $\varphi \in \mathcal{H}$ with $\|\varphi\| = 1$ (i.e. ϱ is a rank-one projection). Prove that $\operatorname{tr}(|\gamma - \varrho|) \leqslant 2\|\gamma - \varrho\|_2$, where $\|\cdot\|_2$ denotes the Hilbert-Schmidt norm.

Remark: This estimate is of great importance in mean-field theory because it allows one to control the effect of rank-one perturbations in the trace norm by the weaker Hilbert-Schmidt norm, which is induced by a scalar product.

Exercise 15

Let $\Psi \in \mathcal{H}_1 \otimes \mathcal{H}_2$ and suppose that there exists $\varphi \in \mathcal{H}_1$ such that

$$\langle \Psi, (B \otimes \mathbf{1}_{\mathcal{H}_2})\Psi \rangle = \langle \varphi, B\varphi \rangle$$

for all $B \in \mathcal{L}(\mathcal{H}_1)$. Prove that this is only possible if Ψ is a product state, i.e. $\Psi = \varphi \otimes \eta$ for some $\eta \in \mathcal{H}_2$ with $\|\eta\| = 1$.

Remark: This exercise shows that it is impossible to realize the partial trace on the level of the wave function: If $\Gamma = |\Psi\rangle\langle\Psi|$, then the partial trace $\gamma = \gamma_\Gamma$ is not necessarily a rank-one projection, too.