

Summer Term 2013

Mathematical Physics

5th Problem sheet

Problem 14

Let $V \in L^\infty$ be a negative potential decaying at infinity and assume that there exist $C, a > 0$ such that

$$V(x) \leq -C|x|^{-2+\varepsilon} \quad \text{for } |x| > a.$$

Prove that $-\Delta + V(x)$ has infinitely many negative eigenvalues.

Problem 15

Let $\Omega \subset \mathbb{R}^d$ be a connected domain with C^1 -boundary $\partial\Omega$ and let Δ_Ω denote the Laplacian on Ω with Dirichlet boundary conditions. Show that $\sigma(\Delta_\Omega) = \sigma_d(\Delta_\Omega)$.