

Partial Differential Equations II

Exercise Sheet 3

Exercise 1

Let $u \in (C^{1,\alpha} \cap C^2)(\mathbb{R}^n)$, $f \in (C^{0,\alpha} \cap C^1)(\mathbb{R}^n, \mathbb{R}^n)$ and $g \in (C^0 \cap L^p)(\mathbb{R}^n)$, where $0 < \alpha < 1$ and $1 - \frac{n}{p} = \alpha$. Assume u is a solution of the equation

$$\Delta u = \operatorname{div} f + g \quad \text{in } \mathbb{R}^n.$$

Show that there is a constant $C = C(n, \alpha)$ such that

$$[Du]_{\alpha, \mathbb{R}^n} \leq C ([f]_{\alpha, \mathbb{R}^n} + \|g\|_{L^p(\mathbb{R}^n)}).$$