

Partial Differential Equations II

Exercise Sheet 6

Exercise 1

For $n \geq 2$, $0 < \lambda < 1$, consider the Dirichlet problem

$$\begin{aligned} \Delta u + \left(\frac{n-1}{1-\lambda} - 1 \right) \frac{x_i x_j}{|x|^2} \partial_{ij} u &= 0 \quad \text{in } B_1(0), \\ u &= 0 \quad \text{on } \partial B_1(0). \end{aligned}$$

Show that $v(x) := |x|^\lambda - 1$ is a solution of this problem. Moreover, show that

$$v \in W^{2,p}(B_1(0)) \setminus W^{2,p_0}(B_1(0)) \quad \text{for } p < p_0 := \frac{n}{2-\lambda}$$

and conclude that Theorem III.1 is not true for arbitrary ε .

Exercise 2

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain. Let $u \in W_0^{1,2}(\Omega)$ be a weak solution of

$$-\Delta u = |u|^{p-1}u, \quad \text{where } 1 < p < \frac{n+2}{n-2}.$$

Show that $u \in C_{\text{loc}}^{0,\alpha}(\Omega)$.