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Mathematical Methods in Quantum Mechanics I

1st Exercise Sheet

Exercise 1:
Suppose $\mathfrak{D}$ is a vector space. Let $s(f,g)$ be a sesquilinear form on $\mathfrak{D}$ and $q(f) = s(f,f)$ the associated quadratic form. Prove the *parallelogram law*

$$q(f+g) + q(f-g) = 2q(f) + 2q(g) \tag{1}$$

and the *polarization identity*

$$s(f,g) = \frac{1}{4}\left[q(f+g) - q(f-g)\right] + \frac{i}{4}\left[q(f-ig) - q(f+ig)\right]. \tag{2}$$

Show that $s(f,g)$ is symmetric if and only if $q(f)$ is real-valued.

Exercise 2:
Let $A : \mathcal{D}(A) \subset X \to X$ be a linear operator such that $\langle \psi, A\psi \rangle = \langle A\psi, \psi \rangle$ for all $\psi \in \mathcal{D}(X)$. Then

$$\langle \varphi, A\psi \rangle = \langle A\varphi, \psi \rangle$$

for all $\varphi, \psi \in \mathcal{D}(X)$.
Hint: Use the polarization identity.

Exercise 3:
Show the following:

1. Let operator $A : \mathcal{D}(A) \subset X \to X$ be linear and not closed. Then $\sigma(A) = \mathbb{C}$.
2. Let $A : \mathcal{D}(A) = X \to X$ be a closed linear operator. Then A is bounded.
   Hint: Use the closed graph theorem.

Exercise 4:
Let $A : \mathcal{D}(A) \subseteq X \to X$ be a closed linear operator and $z, w \in \rho(A)$. Then

1. $R_A(z) - R_A(w) = (w-z)R_A(z)R_A(w)$.
2. $R_A(z)R_A(w) = R_A(w)R_A(z)$.
3. $R_A(z)A \subseteq AR_A(z) = zR_A(z) - I$.