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Mathematical Methods in Quantum Mechanics I

3rd Exercise Sheet

Exercise 9:
Let $V \in C(\mathbb{R}^n; \mathbb{C})$. Then we define $M_V : \mathcal{D}(M_V) \subseteq L^2(\mathbb{R}^n) \to L^2(\mathbb{R}^n)$ as

$$M_V f(x) = V(x) f(x)$$

where $D(M_V) := \{f \in L^2(\mathbb{R}^n) : V \times f \in \L^2(\mathbb{R}^n)\}$. Show that

$$\overline{\mathrm{Ran}(V)} \subseteq \sigma(M_V)\,.$$

Exercise 10:
Let A, B be densely defined linear operators on a Hilbert space $\mathcal{H}$. Show the following

$$A \subset B \Rightarrow B^* \subset A^*\,.$$

Exercise 11:
Let $\mathcal{H} = L^2(\mathbb{R})$ and $g \in \mathcal{H}$ with $\|g\| = 1$. Let $A : C_c(\mathbb{R}) \subset \mathcal{H} \to \mathcal{H}, Af = f(0)g$. Show that

$$\langle \varphi, g \rangle \neq 0, \varphi \in \mathcal{H} \Rightarrow \varphi \notin \mathcal{D}(A^*)\,.$$