

Mathematical Methods in Quantum Mechanics I

6th Exercise Sheet

Exercise 18:

Let $U(t) \in \mathcal{L}(L^2(\mathbb{R}))$ be defined as $(U(t)\psi)(x) = \psi(x - t)$. Find the generator of $U(t)$ and describe its domain explicitly.

Exercise 19:

Let \mathcal{H} be a Hilbert space and A, B, C be bounded linear operators on \mathcal{H} . Show the following statements:

1. $[AB, C] = A[B, C] + [A, C]B$,
2. $[A, BC] = B[A, C] + [A, B]C$,
3. $[A, [B, C]] + [B, [C, A]] + [C, [A, B]] = 0$.

Exercise 20:

Let \mathcal{H} be a Hilbert space and H be a densely defined self-adjoint operator on \mathcal{H} . Furthermore define $B_n := in(H - in)^{-1}$ and $H_n := B_{-n}HB_n$. Show the following statements:

1. $[H_n, H_m] = 0$,
2. $[H_n, \exp(itH_m)] = 0$.