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Mathematical Methods in Quantum Mechanics I

7th Exercise Sheet

Exercise 21:
Consider Hilbert space $L^2(\mathbb{R})$. Find all states that minimize uncertainty relations for momentum operator p and position operator x, i.e.

$$\Delta x_\psi \Delta p_\psi = \frac{1}{4}|\langle \psi[x,p]\psi\rangle|^2 .$$

For simplicity assume $\langle \psi, x\psi\rangle = 0$ and $\langle \psi, \partial_x \psi\rangle = 0$.

Exercise 22:
Let $\mathcal{H}$ be a Hilbert space, D its dense subset and x_n be a bounded sequence. Show that the following are equivalent

$$x_n \rightharpoonup x \Leftrightarrow \forall v \in D : \langle x_n, v\rangle \to \langle x, v\rangle .$$

Exercise 23:
Let $\mathcal{H} \neq \{0\}$ be a Hilbert space and $A \in \mathcal{L}(H)$. Show that the spectrum of A is not empty.

Exercise 24:
Let $\mathcal{H}$ be a Hilbert space and $A \in \mathcal{L}(H)$. Assume that for all $|s| < r$ the function $f(s) := (1 - sA)^{-1}$ exists and is bounded. Prove that $f(s)$ has a series expansion in s and that for all $\epsilon > 0$ the series converges uniformly in $|s| < r - \epsilon$.