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# Spectral theory

## 1. Exercise Sheet

### Exercise 1 (Closed Operators)

1. Show that every separable Hilbertspace $H$ has a finite or countable orthonormal basis.
2. We consider on the Hilbertspace $H = L^2(0,1)$ the linear operator $T$ acting on the Sobolevspace $D(T) = H^1(0,1)$ by $Tf = \mathrm{i}f'$. Is the operator $T$ closed? / symmetric? / self-adjoint? / semi-bounded from below?
3. Now we consider again the Hilbertspace $H = L^2(0,1)$ and the linear operator $\widetilde{T}$ acting on the Sobolevspace $D\left(\widetilde{T}\right) = H_0^1(0,1)$ by $\widetilde{T}f = \mathrm{i}f'$. Is the operator $\widetilde{T}$ closed? / symmetric? / self-adjoint? / semi-bounded from below?

### Exercise 2 (Multiplication operators are closed)

Consider the Hilbertspace $H = L^2\left(\mathbb{R}^d\right)$ and fix some function $f \in L^\infty_{\mathrm{loc}}\left(\mathbb{R}^d\right)$. We introduce the multiplication operator $M_f$ acting on

$$D\left(M_f\right) = \left\{u \in L^2\left(\mathbb{R}^d\right) : fu \in L^2\left(\mathbb{R}^d\right)\right\} \text{ by } M_f u = fu.$$

Show that the linear operator $M_f$ is closed in $H$.

#### Exercise Sheets

Sometimes there will be an exercise sheet, but perhaps not every week, which you can find on the webpage. You can hand in your solution of every exercise and I (Michael) will correct it. Please deliver your solution in the box on the ground floor of the math building or in the problem class. In the problem class we will discuss the solutions and do some additional stuff.
If I have not enough time, I will upload my sketches to the missing problems on the webpage.
Feel free to ask questions or come to my office.