12.05.2017
Prof. Dr. Tobias Lamm
M.Sc. Michael Ullmann

Karlsruhe Institute of Technology

Spectral theory

2. Exercise Sheet

Exercise 1 (Proof of Lemma IV.2)

1. (Lemma of Fekete) If $(a_n)_{n\in\mathbb{N}} \subseteq [0,\infty)$ is a real sequence with $0 \leq a_{n+m} \leq a_n \cdot a_m$ for all $n, m \in \mathbb{N}$, then there holds
$$\lim_{n\to\infty} a_n^{\frac{1}{n}} = \inf_{n\in\mathbb{N}} a_n^{\frac{1}{n}}.$$

2. We define for a bounded operator $A \in L(X)$ the spectral radius
$$r(A) := \sup\{|\lambda| : \lambda \in \sigma(A)\}.$$
Show the following:
$$r(A) = \lim_{n\to\infty} \|A^n\|_{L(X)}^{\frac{1}{n}}.$$

3. Let $A \in L(X)$ be a bounded operator and $\lambda \in \mathbb{C}$ such that $|\lambda| > r(A)$. Show that
$$R(\lambda, A) = \sum_{n=0}^{\infty} \lambda^{-(n+1)} A^n.$$

4. Let $A \in L(X)$ be a bounded operator. Assume that $\lambda \in \mathbb{C}$ with $|\lambda| > \delta \|A\|_{L(X)}$ for some $\delta > 1$, show in this case the estimate
$$\|\lambda R(\lambda, A)\|_{L(X)} < \frac{\delta}{\delta - 1}.$$

Exercise 2 (Dunford Lemma)

Show that weakly analytic is equivalent to analytic.

Exercise Sheets

Sometimes there will be an exercise sheet, but perhaps not every week, which you can find on the webpage. You can hand in your solution of every exercise and I (Michael) will correct it. Please deliver your solution in the box on the ground floor of the math building or in the problem class. In the problem class we will discuss the solutions and do some additional stuff.
If I have not enough time, I will upload my sketches to the missing problems on the webpage.
Feel free to ask questions or come to my office.