

Spectral Theory

3rd Exercise Sheet

Exercise 10:

Let A be a linear operator in X and let $X_0 \subseteq X$ be a closed linear subspace invariant under the resolvents of A . Define $A_0 := A|_{\mathcal{D}(A_0)}$ where $\mathcal{D}(A_0) = \{x \in X_0 \cap \mathcal{D}(A), Ax \in X_0\}$. Show that $\rho(A) \subseteq \rho(A_0)$.

Exercise 11:

Let $X = C([0, 1])$, $m \in X$ and $T : X \rightarrow X$ defined by $Tf = mf$. Determine $\sigma_p(T)$, $\sigma_c(T)$ and $\sigma_r(T)$.

Exercise 12:

Show that the Fourier transform $\mathcal{F} : \mathcal{S}(\mathbb{R}^d) \rightarrow \mathcal{S}(\mathbb{R}^d)$ is well defined automorphism.