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# Spectral Theory

## 4th Exercise Sheet

**Exercise 13:**
Let $k \in L^1(\mathbb{R}^d)$. Show that the operator $T : L^2(\mathbb{R}^d) \to L^2(\mathbb{R}^d)$ defined as

$$Tf := k \star f, \quad f \in L^2(\mathbb{R}^d)$$

is well defined, linear and bounded. Determine its spectrum $\sigma_p(T)$, $\sigma_c(T)$ and $\sigma_r(T)$.

**Exercise 14:**
Show that for $f, \hat{f} \in L^1(\mathbb{R}^d)$ Fourier inversion formula

$$f(x) = \int_{\mathbb{R}^d} e^{2\pi i x\xi} \hat{f}(\xi) \mathrm{d}\xi \quad \text{for a. e. } x \in \mathbb{R}^d$$

holds. Furthermore show that $f, \hat{f} \in C_0(\mathbb{R}^d)$ when modified suitably on a null-set.

**Exercise 15: Heisenberg uncertainty principle**
Let $d = 1$. Then for arbitrary $\psi \in S(\mathbb{R})$ the inequality

$$\|x\psi\|_2 \|\xi\hat{\psi}\|_2 \geq \frac{1}{4\pi} \|\psi\|_2^2 \, .$$

holds.
*Hint*: Look at the expression $2\,\mathrm{Re}\langle x\psi, \psi'\rangle$.