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Spectral Theory

6th Exercise Sheet

Exercise 19:
Let X, Y, Z be Banach spaces, $T \in \Phi(X, Y)$ and $S \in \Phi(Y, Z)$. Show that $ST \in \Phi(X, Z)$ and $\mathrm{ind}(ST) = \mathrm{ind}(S) + \mathrm{ind}(T)$.
Hint: First find a complement N of $N(T)$ in $N(ST)$ and a complement N_S of $T(N)$ in $N(S)$. Then represent X and Y as direct sums.

Exercise 20: Show the following:

Let A be a linear operator in $\mathbb{C}^n$ given by $A = \begin{pmatrix} \lambda & 1 & & \\ & \ddots & \ddots & \\ & & \lambda & 1 \\ & & & \lambda \end{pmatrix}$ where $\lambda \in \mathbb{C}$. Find a representation for the operator $f(A)$ (given by the Dunford functional calculus), where f is holomorphic on a neighborhood of λ.

Exercise 21:
Let $X = L^2(\mathbb{R}^+)$ and let $T : X \to X$ be defined by

$$Tf(x) := \frac{1}{x} \int_0^x f(y)dy \quad \text{for} \quad f \in X, x \in \mathbb{R}^+$$

Show that:

1. T is well defined, linear and continuous.
2. T is not compact.
   Hint: You can use Fredholm alternative.