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Spectral Theory

7th Exercise Sheet

Exercise 22:
Show the following:
Let A be a closed linear operator on a Banach Space X and

$$\mathcal{D}(A^{k+1}) := \{x \in \mathcal{D}(A^k) : Ax \in \mathcal{D}(A^k)\}\,.$$

Then $\mathcal{D}(A^k)$ is a Banach space for the norm

$$|||x||| := \sum_{j=0}^{k} \|A^j x\|\,.$$

Exercise 23:
Show the following:

1. Let A be a closed linear operator on a Banach Space X s.t. $\rho(A) \neq \emptyset$. Then A^k is closed for any $k \in \mathbb{N}$.
2. In particular, the graph norm of A^k is equivalent to the norm of Ex. 22.

Exercise 24:
Let A be a closed linear operator on a Banach Space X. Then

$$\sigma(A^k) = \{\lambda^k : \lambda \in \sigma(A)\}\,.$$

Exercise 25:
Let $1 \leq p < \infty$ and $X \times Y$ be σ-finite product measure space with measures dx and dy respectively. Then for a measurable function $F(x, y)$

$$\left(\int_Y \left(\int_X |F(x,y)|\mathrm{d}x\right)^p \mathrm{d}y\right)^{\frac{1}{p}} \leq \int_X \left(\int_Y |F(x,y)|^p \mathrm{d}y\right)^{\frac{1}{p}} \mathrm{d}x$$

holds. Furthermore equality holds for $p = 1$ or $F(x, y) = \phi(x)\psi(y)$ if $p > 1$.