Institut für Analysis
PD Dr. Peer Christian Kunstmann
Dr. Michal Jex

SS2018/19
24.6.2019

Spectral Theory

8th Exercise Sheet

Exercise 26:
Show the following:
Let $B \in S_2(H)$. Then $B^* \in S_2(H)$ and furthermore

$$\|S\|_{\nu_2}^2 = \sum_{j=1}^{\infty} \|Be_j\|^2 < \infty$$

for any orthonormal basis $E = (e_n) \subseteq H$ with the sum independent of E.

Exercise 27:
Show the following:

1. $S_2(H)$ is a two–sided $\star$–ideal in $\mathcal{L}(H)$ and $\|\cdot\|_2$ is a norm on it which fulfils
$$\|S\|_{\nu_2} \geq \|S\|_2$$
for all $S \in S_2(H)$. Recall that Hilbert–Schmidt operator is compact, i.e. $S_2(H) \subseteq \mathcal{K}(H)$.
2. $S_2(H)$ equipped with the inner product
$$(B|C)_2 = \sum_{j=1}^{\infty} (Be_j|Ce_j)_H$$
is a Hilbert space.

Exercise 28:
Show the following:
Any trace–class operator S is Hilbert–Schmidt, and therefore compact, $S_1(H) \subseteq S_2(H) \subseteq \mathcal{K}(H)$ and

$$\mathrm{Tr}(|S|) \geq \|S\|_{\nu_2} \geq \|S\|_2 \,.$$