Karlsruher Institut für Technologie

Institut für Analysis
PD Dr. Peer Christian Kunstmann
Dr. Michal Jex

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Spectral Theory

10th Exercise Sheet

Exercise 33:
Let A be a self-adjoint positive operator. Then we define for $t \in \mathbb{R}^+$

$$T(t) := e^{-tA} .$$

Show that

1. $T(t)$ is self-adjoint.
2. $T(t)$ is a strongly continuous semigroup, i.e. $T(t)T(s) = T(t+s)$ for every $s, t \in \mathbb{R}^+$.
3. Furthermore let $x \in \mathcal{D}(A)$ and $Ax = y$. Then for every $s > 0$

$$\lim_{t \to 0_+} \frac{1}{t}(T(s)x - T(t+s)x) = T(s)y$$

   holds.

Exercise 34:
Let $T \in \mathcal{L}(H)$ be self-adjoint. Show or calculate the following:

1. Let $\sigma(T) = \{0, 1\}$, then T is orthogonal projector,
2. $\|TR(\lambda, T)^2\|_{\mathcal{L}(H)}$ for $\lambda \in \rho(T)$,
3. $\|Te^{-sT^2}\|_{\mathcal{L}(H)}$ for $s \geq 0$,
4. Assume that T is positive $\|(I + sT)^{-1}e^{isT}\|_{\mathcal{L}(H)}$ for $s \geq 0$.

Exercise 35:
Let $A, B \in \mathcal{L}(H)$. Then

$$e^{A+B} = \lim_{N \to \infty} \left(e^{A/N} e^{B/N}\right)^N .$$