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Spectral Theory

12th Exercise Sheet

Exercise 40:
Let A be densely defined and symmetric in Hilbert space H and $(Au|u) \geq 0$ for all $u \in \mathcal{D}(A)$. Define

$$\mathfrak{a} := \mathcal{D}(A) \times \mathcal{D}(A) \to \mathbb{C}, \mathfrak{a}(u,v) = (Au|v)\,.$$

Show that $\mathfrak{a}$ is closable.

Exercise 41:
Let $\Omega = B(0,1) \subseteq \mathbb{R}^2$ and let $a := \begin{pmatrix} 1 & -\beta \\ \beta & 1 \end{pmatrix}$ where $\beta \in \mathbb{R}$ is fixed. Define

$$\mathfrak{a}(u,v) := \int_\Omega (a\nabla u) \cdot \overline{\nabla v} \mathrm{d}x\,.$$

where $u, v \in V = W^{1,2}(\Omega)$. Let A be the operator associated with $\mathfrak{a}$. Calculate Au and the corresponding boundary conditions for $u \in \mathcal{D}(A) \cap C^2(U)$ where U is an open set such that $\overline{\Omega} \subseteq U$.

Exercise 42:
Let $\lambda_1, \ldots, \lambda_n$ be the eigenvalues of an $n \times n$ self-adjoint matrix A, written in increasing order as usual. Show that for any $m \leq n$ one has

$$\sum_{r=1}^{m} \lambda_r = \min\{\mathrm{tr}(L) : \dim(L) = m\}\,,$$

where L denotes any linear subspace $\mathbb{C}^n$, and

$$\mathrm{tr}(L) := \sum_{r=1}^{m} (Ae_r|e_r)\,.$$

for some (any) orthonormal basis $\{e_r\}_{r=1}^m$ of L.