Exercise 40:
Let $A$ be densely defined and symmetric in Hilbert space $H$ and $(Au|u) \geq 0$ for all $u \in D(A)$. Define
\[ a := D(A) \times D(A) \rightarrow \mathbb{C}, \ a(u,v) = (Au|v). \]
Show that $a$ is closable.

Exercise 41:
Let $\Omega = B(0,1) \subseteq \mathbb{R}^2$ and let $a := \begin{pmatrix} 1 & -\beta \\ \beta & 1 \end{pmatrix}$ where $\beta \in \mathbb{R}$ is fixed. Define
\[ a(u,v) := \int_{\Omega} (a \nabla u) \cdot \nabla v \, dx. \]
where $u, v \in V = W^{1,2}(\Omega)$. Let $A$ be the operator associated with $a$. Calculate $Au$ and the corresponding boundary conditions for $u \in D(A) \cap C^2(U)$ where $U$ is an open set such that $\overline{\Omega} \subseteq U$.

Exercise 42:
Let $\lambda_1, \ldots, \lambda_n$ be the eigenvalues of an $n \times n$ self-adjoint matrix $A$, written in increasing order as usual. Show that for any $m \leq n$ one has
\[ \sum_{r=1}^{m} \lambda_r = \min \{ \text{tr}(L) : \dim(L) = m \}, \]
where $L$ denotes any linear subspace $\mathbb{C}^n$, and
\[ \text{tr}(L) := \sum_{r=1}^{m} (Ae_r|e_r). \]
for some (any) orthonormal basis $\{e_r\}_{r=1}^{m}$ of $L$.  

http://www.math.kit.edu/iana1/edu/spectraltheo2019s/en