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Best Constant in Sobolev Inequality (*).

GIORGIO TALENTI (Firenze)

Summary. – *The best constant for the simplest Sobolev inequality is exhibited. The proof is accomplished by symmetrizations (rearrangements in the sense of Hardy-Littlewood) and one-dimensional calculus of variations.*

0. – The main result of the present paper is the following:

THEOREM. – *Let u be any real* (or complex) *valued function, defined on the whole m-dimensional euclidean space R^m, sufficiently smooth and decaying fast enough at infinity. Moreover let p be any number such that:* $1<p<m$. *Then:*

$$\left\{\int_{R^m}|u|^q\,dx\right\}^{1/q}\leqslant C\left\{\int_{R^m}|Du|^p\,dx\right\}^{1/p}, \tag{1}$$

where: $|Du|$ *is the length of the gradient* Du *of* u, $q=mp/(m-p)$ *and*

$$C=\pi^{-\frac{1}{2}}m^{-1/p}\left(\frac{p-1}{m-p}\right)^{1-1/p}\left\{\frac{\Gamma(1+m/2)\Gamma(m)}{\Gamma(m/p)\Gamma(1+m-m/p)}\right\}^{1/m} \tag{2}$$

The equality sign holds in (1) *if* u *has the form:*

$$u(x)=\left[a+b|x|^{p/(p-1)}\right]^{1-m/p}, \tag{3}$$

where $|x|=(x_1^2+\ldots+x_m^2)^{\frac{1}{2}}$ *and* a, b *are positive constants.*

Sobolev inequalities, also called Sobolev imbedding theorems, are very popular among writers in partial differential equations or in the calculus of variations, and have been investigated by a great number of authors. Nevertheless there is a question concerning Sobolev inequalities, which seems well-known only to a restricted number of specialists working in geometric measure theory. The question is the connection between Sobolev inequalities and the classical isoperimetric inequality for subsets of euclidean spaces. Our aim is to advertise such a connection.

To be specific, we are concerned with the simplest Sobolev inequality

$$\|u\|_{L^q(R^m)}\leqslant(\text{constant independent of } u)\|Du\|_{L^p(R^m)}, \tag{4}$$

(*) Entrata in Redazione il 16 luglio 1975.