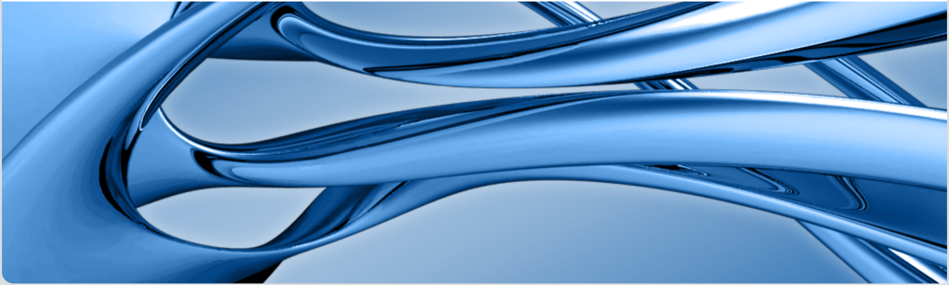
Karlsruhe Institute of Technology

Communication in wireless ad hoc networks

a stochastic geometry model

Institute for Applied and Numerical Mathematics – RG Numerical Simulation, Optimization and High Performance Computing



Introduction

Ad hoc networks

- No (hard-wired) infrastructure
- Self-organization
- Cooperation among the subscribers ($\equiv$ nodes)

Applications

- Military and law enforcement communication
- Rapidly deployed temporary networks
- Sensor networks

Modeling

- Avoid expensive mistakes
- Test for suitability
- Optimize system parameters

Considered scenario

Inhomogeneous Mobile ad hoc network Nearest Receiver (IMNR)

- **Node positions:** finite, inhomogeneous Poisson point process
- **Channel access method:** slotted ALOHA
- **Routing:** nearest receiver
- **Transmission success:** signal-to-noise ratio condition
- **Transmit power:** constant
- **Path loss model:** omnidirectional power law

Performance metrics

- **Local:** probability of coverage
- **Global:** mean number of successful transmissions

Classification & literature survey

Literature survey

- **Baccelli and Błaszczyszyn (2009):** MNR model
  - **Node positions:** *homogeneous* PPP
  - **Performance metrics:** probability of coverage & density of successful transmissions
- **Haenggi and Ganti (2008):** analysis of interference
  - **Node positions:** Poisson cluster processes & general motion-invariant PP
  - **Performance metric:** probability of coverage
- **Haenggi, Andrews, Baccelli et al. (2009):** homogeneous case
- **Weber, Andrews and Jindal (2010):** Transmission Capacity

Classification

- *inhomogeneous, finite* PPP
- *global* metric of performance

Structure

- System model
- Performance metrics
- Results
- Outlook

Network geometry & channel access

PP of node positions Φ

- Φ finite, inhomogeneous PPP on $\mathbb{R}^2$
- $\Lambda = \lambda G$ intensity measure (IM)
  - $\lambda > 0$ expected total number of nodes
  - G distribution of the position of a typical node
  - g probability density function (PDF) of G

PPs of roles Φ_R, transmitters Φ_{Tx} and receivers Φ_{Rx}

- Φ_R independent F-marking of Φ
  - $F = p_{MAC}\delta_1 + (1 - p_{MAC})\delta_0$
  - $p_{MAC} \in (0, 1)$ probability of media access
- $\Phi_{Tx} = \Phi_R(\cdot \times \{1\})$ PP of transmitters
  - IM: $\Lambda_{Tx} = p_{MAC}\Lambda = \lambda_{Tx}G$
- $\Phi_{Rx} = \Phi_R(\cdot \times \{0\})$ PP of receivers, IM: $\Lambda_{Rx} = \lambda_{Rx}G$
- Φ_{Tx}, Φ_{Rx} independent PPPs

Routing Model

Definition: nearest neighbour function Y^*

- $x \in \mathbb{R}^2$ (position of the transmitter)
- $\mu = \sum_{i=1}^{l} \delta_{y_i}$ boundedly finite counting measure on $\mathbb{R}^2$ (realization of $\Phi_{\mathrm{Rx}} \equiv$ positions of receivers)
- Define $Y^* : \mathbb{R}^2 \times N_{\mathbb{R}^2} \to \overline{\mathbb{R}^2}$ by:

$$Y^*(x, \mu) = \begin{cases} \underset{y \in \mu}{\operatorname{argmin}}\{\|x - y\|\} & \text{if existent and unique} \\ \infty & \text{otherwise} \end{cases}$$

PP of transmitter-receiver pairs Φ_{TxRx}

- $\Phi_{\mathrm{Tx}} = \sum_{i=1}^{l} \delta_{X_i} \equiv$ Positions of transmitters
- $Y_i = Y^*(X_i, \Phi_{\mathrm{Rx}}) \equiv$ targeted receiver of the transmitter at X_i
- $\Phi_{\mathrm{TxRx}} = \sum_{i=1}^{l} \delta_{(X_i, Y_i)} \equiv$ PP of transmitter-receiver pairs

Success of a transmission

signal-to-interference ratio

$$\mathrm{SIR} = \frac{\ell(\|x-y\|)}{\int \ell(\|y-z\|)\nu(dz)} = \frac{\|x-y\|^{-\alpha}}{\int \|y-z\|^{-\alpha}\nu(dz)}$$

- $\ell = \ell(r)$ omnidirectional path loss
  - $\ell(r) = r^{-\alpha}$ power law where $2 \leq \alpha \leq 4$ loss exponent
- x position of the transmitter
- y position of the receiver
- $\nu = \sum_{i=1}^{I} \delta_{z_i}$ positions of competing transmitters

SIR success function s

Define $s: \mathbb{R}^2 \times \overline{\mathbb{R}^2} \times N_{\mathbb{R}^2} \to \{0, 1\}$ by:

$$s(x, y, \nu) = \mathbb{1}_{\{\mathrm{SIR} \geq \beta\}}(x, y, \nu)$$

Structure

Done: system model

- **Network geometry:** inhomogeneous PPP
- **Channel access method:** slotted ALOHA
- **Routing model:** nearest receiver routing
- **Success of a transmission:** SIR success function

Next: performance metrics

- **Local performance metric:** probability of coverage
- **Global performance metric:** mean number of successful transmissions

Local performance metric

probability of coverage $p_c(x, y)$

$$p_c(x, y) = \int s(x, y, \nu(\cdot \times \overline{\mathbb{R}^2}))\mathbb{P}^{!\mathrm{TxRx}}_{(x,y)}(d\nu)$$

- $x \in \mathbb{R}^2 \equiv$ position of the transmitter
- $y \in \overline{\mathbb{R}^2} \equiv$ position of the targeted receiver
- s success function
- $\mathbb{P}^{!\mathrm{TxRx}}_{(x,y)}$ local modified Palm distribution of Φ^{TxRx} at $(x, y) \equiv$ distribution of the PP of transmissions competing with (x, y)
- $\nu(\cdot \times \overline{\mathbb{R}^2}) \equiv$ positions of interferers (do not care about positions of the corresponding receivers)

Global performance metric

Mean number of successful transmissions T

$$T = \mathbb{E} \int s(x, Y^*(x, \Phi^{\mathrm{Rx}}), \Phi^{\mathrm{Tx}} \setminus x) \Phi^{\mathrm{Tx}}(dx)$$

- s success function
- $x \equiv$ position of the transmitter
- $y = Y^*(x, \Phi^{\mathrm{Rx}})$ position of the receiver
  point of Φ^{Rx} nearest to x
- $\nu = \Phi^{\mathrm{Tx}} \setminus x \equiv$ positions of interferers

Structure

Done: performance metrics

- **Local performance metric:** probability of coverage
- **Global performance metric:** mean number of successful transmissions

Next: Results

- Uniqueness of the nearest receiver
- Distribution of the nearest receiver
- Estimate of the probability of coverage
- Formula for the global performance metric

Uniqueness of the nearest receiver

$$Y^*(x, \Phi^{\text{Rx}}) = \begin{cases} \underset{y \in \Phi^{\text{Rx}}}{\text{argmin}}\{\|x - y\|\} & \text{if existent and unique} \\ \infty & \text{otherwise} \end{cases}$$

- $x \equiv$ position of the transmitter
- $Y^*(x, \Phi^{\text{Rx}}) \equiv$ targeted receiver of the transmitter at x
- $\Phi^{\text{Rx}} \equiv$ positions of the receivers

Theorem

It is:

- $\mathbb{P}\left(Y^*(x, \Phi^{\text{Rx}}) = \infty\right) = \mathbb{P}\left(\Phi^{\text{Rx}} = \emptyset\right) = e^{-\lambda_{\text{Rx}}}$
- $\mathbb{P}\left(Y^*(x, \Phi^{\text{Rx}}) = \infty \,|\, \Phi^{\text{Rx}} \neq \emptyset\right) = 0$

Distribution of the nearest receiver

Theorem: distribution of the distance to the nearest receiver

Distributions of $\left\|x - Y^*(x, \Phi^{\mathrm{Rx}})\right\|$ form a stochastic kernel H with source $\mathbb{R}^2$ and target $\overline{\mathbb{R}}$. For all $x \in \mathbb{R}^2$:

- $H(x, \cdot) = e^{-\lambda_{\mathrm{Rx}}} \delta_\infty(\cdot) + H_{\mathrm{ac}}(x, \cdot)$
- The PDF of $H_{\mathrm{ac}}(x, \cdot)$ is given by:
  $h(x, r) = \lambda_{\mathrm{Rx}} \int_{\partial B_r(x)} g \, d\sigma \cdot e^{-\lambda_{\mathrm{Rx}} G(\overline{B}_r(x))}$

Theorem: distribution of nearest receiver

Distributions of $Y^*(x, \Phi_{\mathrm{Rx}})$ form a stochastic kernel G^* with source $\mathbb{R}^2$ and target $\overline{\mathbb{R}^2}$. For all $x \in \mathbb{R}^2$:

- $G^*(x, \cdot) = e^{-\lambda_{\mathrm{Rx}}} \delta_\infty(\cdot) + G^*_{\mathrm{ac}}(x, \cdot)$
- The PDF of $G^*_{\mathrm{ac}}(x, \cdot)$ is given by:
  $g^*(x, y) = \lambda_{\mathrm{Rx}} g(y) e^{-\lambda_{\mathrm{Rx}} G(\overline{B}_{\|x-y\|}(x))} = g(y \mid \|x - y\| = r) h(x, r)$

Estimate of the probability of coverage

$$p_c(x,y) = \int s(x,y,\nu(\cdot \times \overline{\mathbb{R}^2}))\mathbb{P}^{!\mathrm{TxRx}}_{(x,y)}(d\nu) = \int s(x,y,\nu)\mathbb{P}^{\mathrm{Tx}}(d\nu)$$

- Complicated dependence on ν:

$$s(x,y,\nu) = 0 \Leftrightarrow \frac{\|x-y\|^{-\alpha}}{\int \|y-z\|^{-\alpha}\nu(dz)} < \beta$$

- Estimate of $s(x,y,\nu)$: dominant interferers (Weber et al.)
- Consider only the case, that one interferer is enough to disturb the transmission:

$$\Rightarrow \|y-z\| < \beta^{\frac{1}{\alpha}} \|x-y\|$$

- $p_c(x,y) \leq \mathbb{P}\left(\Phi^{\mathrm{Tx}}(B_{\beta^{\frac{1}{\alpha}}\|x-y\|}(y)) = 0\right) = e^{-\lambda_{\mathrm{Tx}} G(B_{\beta^{\frac{1}{\alpha}}\|x-y\|}(y))}$

Formula for the global performance metric

$$
\begin{aligned}
T &= \mathbb{E} \int s(x, Y^*(x, \Phi^{\mathrm{Rx}}), \Phi^{\mathrm{Tx}} \setminus x) \Phi^{\mathrm{Tx}}(dx) \\
&\overset{\Phi_{\mathrm{Tx}}, \Phi_{\mathrm{Rx}}}{\underset{\text{indep.}}{=}} \iiint s(x, Y^*(x, \mu), \nu \setminus x) \nu(dx) \mathbb{P}^{\mathrm{Rx}}(d\mu) \mathbb{P}^{\mathrm{Tx}}(d\nu) \\
&\overset{\text{Tonelli}}{=} \iiint s(x, Y^*(x, \mu), \nu \setminus x) \mathbb{P}^{\mathrm{Rx}}(d\mu) \nu(dx) \mathbb{P}^{\mathrm{Tx}}(d\nu) \\
&\overset{\text{Distribution}}{\underset{\text{of NR}}{=}} \iiint s(x, y, \nu \setminus x) G^*(x, dy) \nu(dx) \mathbb{P}^{\mathrm{Tx}}(d\nu) \\
&= \mathbb{E} \iint s(x, y, \Phi^{\mathrm{Tx}} \setminus x) G^*(x, dy) \Phi^{\mathrm{Tx}}(dx) \\
&\overset{\text{refined}}{\underset{\text{Campbell}}{=}} \iiint s(x, y, \nu) G^*(x, dy) \mathbb{P}_x^{!\mathrm{Tx}}(d\nu) \Lambda_{\mathrm{Tx}}(dx) \\
&\overset{\text{Slivnyak}}{\underset{\text{Tonelli}}{=}} \iiint s(x, y, \nu) \mathbb{P}^{\mathrm{Tx}}(d\nu) G^*(x, dy) \Lambda_{\mathrm{Tx}}(dx) \\
&= \iint p_c(x, y) G^*(x, dy) \Lambda_{\mathrm{Tx}}(dx)
\end{aligned}
$$

Structure

Done: results

- Uniqueness of the nearest receiver
- Distribution of the nearest receiver
- Estimate of the probability of coverage
- Formula for the global performance metric

Next:

- Résumé
- Outlook

Résumé

Comparison with the homogeneous case (MNR Model)

- Finite network ⇒ global performance metric
- Price to pay is higher complexity:
  - $p_c(r) \Rightarrow p_c(x, y)$
  - $G^*(dr) \Rightarrow G^*(x, dy)$

Suitability of the IMNR model for describing sensor networks

- Complete independence of node positions ⇒ PPP
- Simple channel access method ⇒ slotted ALOHA
- Undirected transmissions ⇒ omnidirectional path loss
- Energy Efficiency ⇒ nearest receiver routing (extreme case)

Outlook

Next steps

- Numerical evaluation of the integral formula
- Optimization of the system parameters
- Further refinement of the model:
  - Ambient & thermal noise
  - Fading
  - Other path loss models (fix unphysical singularity at 0)
  - Estimate the global performance metric from below
  - Power control

Big picture

- IMNR: single time slot
  $\Rightarrow$ correlation between time slots
- IMNR: only physical- and data link layer
  $\Rightarrow$ network layer (routing)

Thank you for your attention!

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Questions?

Basic definitions

- $(\mathbb{X}, d_{\mathbb{X}})$ nonempty complete separable metric space
- $\mathcal{B}_{\mathbb{X}}$ Borel-σ-algebra, $\mathcal{B}_{\mathbb{X}}^b$ family of bounded measureable sets
- μ boundedly finite counting measure on $(\mathbb{X}, \mathcal{B}_{\mathbb{X}})$, if

$$\mu(A) \in \mathbb{N}_0$$

  for each $A \in \mathcal{B}_{\mathbb{X}}^b$
- $N_{\mathbb{X}}$ family of boundedly finite counting measures
- $\mathcal{N}_{\mathbb{X}} = \sigma(\mu \mapsto \mu(A);\ A \in \mathcal{B})$ its σ-Algebra
- Φ is a point process (PP), if it is a $(N_{\mathbb{X}}, \mathcal{N}_{\mathbb{X}})$-valued random variable

Conclusion: point processes are random boundedly finite counting measures

Representation of point processes

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Example: binomial process

Let $(X_i)_{i\in\mathbb{N}}$ be i.i.d. on $\mathbb{X}$ and $n \in \mathbb{N}$ be fixed. Then

$$\Phi = \sum_{i=1}^{n} \delta_{X_i}$$

is a point process.

Representation theorem

Each point process Φ has a representation

$$\Phi = \sum_{i=1}^{I} \delta_{X_i}$$

where $I \in \mathbb{N}_0 \cup \{\infty\}$, $(X_i)_{i\in\mathbb{N}} \in \mathbb{X}$ are random variables.

Notation: $\Phi = \{X_1, X_2, \ldots, X_I\}$, X_i are called points of Φ.

Moment measures and Campbell‘s theorem

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Definition: first and second moment measures

First moment measure (Intensity Measure, IM) Λ is defined by

$$\Lambda(A) = \mathbb{E}\Phi(A)$$

for each $A \in \mathcal{B}_{\mathbb{X}}$ (mean number of points in A).
Second moment measure Λ^2 is defined by

$$\Lambda^2(A \times B) = \mathbb{E}\Phi(A)\Phi(B)$$

for each $A \times B \in \mathbb{X}^2$ ($\Rightarrow$ covariance of $\Phi(A)$ and $\Phi(B)$).

Campbell‘s theorem

Let $f : \mathbb{X} \to \mathbb{R}$ be measureable and nonnegative or Λ-integrable. Then:

$$\mathbb{E}\left[\int f(x)\Phi(dx)\right] = \mathbb{E}\sum_{i=1}^{I} f(X_i) = \int f(x)\Lambda(dx)$$

Poisson point processes

Definition

Let Λ be the intensity measure of Φ. If

- $\Phi(A_1), \dots, \Phi(A_n)$ independent for mutually disjoint $A_1, \dots, A_n \in \mathcal{B}_{\mathbb{X}}^b$ (independent increments)
- $\Phi(A)$ has Poisson distribution with parameter $\Lambda(A)$:

$$\mathbb{P}(\Phi(A) = k) = e^{-\Lambda(A)} \frac{\Lambda(A)^k}{k!}$$

  for each $k \in \mathbb{N}_0$ (Poisson statistics)

then Φ is called a Poisson point process (PPP).

Theorem

If Λ is boundedly finite and Φ simple (i.e. $\Phi(\{x\}) \leq 1$ for all $x \in \mathbb{X}$ almost surely w.r.t. $\mathbb{P}$), then:

$$\text{independent increments} \Rightarrow \text{Poisson statistics}$$

Marked point processes

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Definition

A marked point process $\tilde{\Phi}$ on $\mathbb{X}$ with marks in $\mathbb{Y}$ is a point process on $\mathbb{X} \times \mathbb{Y}$, such that $\tilde{\Phi}(\cdot \times \mathbb{Y})$ (ground process) is boundedly finite.

Example: independent F-marking (ALOHA)

Let $\Phi = \sum_{i=1}^{I} \delta_{X_i}$ be a PPP on $\mathbb{X} = \mathbb{R}^2$ with IM Λ, $\mathbb{Y} = \{0, 1\}$ and

$$F = p_{\mathrm{MAC}}\delta_1 + (1 - p_{\mathrm{MAC}})\delta_0 \qquad (0 < p_{\mathrm{MAC}} < 1).$$

Furthermore let $Y_i \in \mathbb{Y}$ be i.i.d. as F, $(Y_i)_{i\in\mathbb{N}}$ independent of Φ. Then

$$\Phi_{\mathrm{R}} = \sum_{i=1}^{I} \delta_{(X_i, Y_i)} \quad \Phi_{\mathrm{Rx}} = \Phi_{\mathrm{R}}(\cdot \times \{0\}) \quad \Phi_{\mathrm{Tx}} = \Phi_{\mathrm{R}}(\cdot \times \{1\})$$

are PPP with IMs $\Lambda_{\mathrm{R}} = \Lambda \otimes F$, $\Lambda_{\mathrm{Rx}} = (1 - p_{\mathrm{MAC}})\Lambda$ and $\Lambda_{\mathrm{Tx}} = p_{\mathrm{MAC}}\Lambda$ respectively. PPs Φ_{Rx}, Φ_{Tx} are independent.

Palm theory

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Definition: modified Campbell measure and Palm kernels

Let Φ be a simple PP with IM Λ. Its modified Campbell measure $C^!$ is defined by

$$C^!(A \times B) = \mathbb{E} \int_A \mathbb{1}_B(\Phi \setminus x)\Phi(dx)$$

for all $A \times B \in \mathcal{B}_{\mathbb{X}} \otimes \mathcal{N}_{\mathbb{X}}$. Furthermore, $C^!$ can be factorized:

$$C^! = \Lambda \otimes \mathbb{P}^!_x$$

The stochastic kernel $\mathbb{P}^!_x$ with source $\mathbb{X}$ and target $N_{\mathbb{X}}$ is called modified Palm kernel.

Refined Campbell‘s theorem

Let $f : \mathbb{X} \times N_{\mathbb{X}} \to \mathbb{R}$ be mb. and nonnegative or $C^!$ integrable. Then:

$$\mathbb{E}\left[\int f(x, \Phi \setminus x)\Phi(dx)\right] = \int f(x,\nu)C^!(d(x,\nu)) = \iint f(x,\nu)\mathbb{P}^!_x(d\nu)\Lambda(dx)$$

Two theorems on PPPs

Theorem of Slivnyak and Mecke

Let Φ be a simple PP with finite IM Λ. Then:

$$\Phi \text{ is PPP} \Leftrightarrow \mathbb{P}_x^! = \mathbb{P}^\Phi \text{ almost everywhere w.r.t. } \Lambda$$

Theorem: second moment measure of a PPP

Let Φ be a PPP with boundedly finite IM Λ. Its second moment measure Λ^2 is:

$$\Lambda^2 = \Lambda \otimes \Lambda + \Lambda(\text{diag}(\cdot))$$