Publications

Kaori Nagato-Plum
Department of Mathematics, Karlsruhe Institute of Technology

Refereed Papers

[1] **M. T. Nakao, N. Yamamoto and K. Nagatou, Numerical Verifications for eigenvalues of second-order elliptic operators, Japan Journal of Industrial and Applied Mathematics, Vol.16 No.3 (1999), pp. 307-320.**
We consider a numerical technique to verify exact eigenvalues and eigenfunctions of second-order elliptic operators in some neighborhood of their approximations. We construct, on the computer, a set which satisfies the hypotheses of Schauder's fixed point theorem for a compact map in a certain Sobolev space, and which therefore contains a solution. Moreover, we propose a method to bound the eigenvalue which has the smallest absolute value. A numerical example is presented.

[2] **K. Nagatou, N. Yamamoto and M. T. Nakao, An approach to the numerical verification of solutions for nonlinear elliptic problems with local uniqueness, Numerical Functional Analysis and Optimization, 20(5 & 6) (1999), pp. 543-565.**
We propose a numerical method to verify the existence and local uniqueness of solutions to nonlinear elliptic equations. We numerically construct a set which satisfies the hypotheses of Banach's fixed point theorem in a certain Sobolev space, and which therefore contains a locally unique solution. By using finite element approximations and constructive error estimates, we calculate a lower bound for the modulus of the eigenvalue with smallest absolute value, to evaluate the norm of the inverse of the linearized operator. Utilizing this bound we derive a verification condition of Newton-Kantorovich type. Numerical examples are presented.

[3] **K. Nagatou, A numerical method to verify the elliptic eigenvalue problems including a uniqueness property, Computing 63 (1999), pp. 109-130.**
We propose a numerical method to enclose eigenvalues and eigenfunctions of second-order elliptic operators, proving also local uniqueness properties. We numerically construct a set containing *eigenpairs* which satisfies the hypotheses of Banach's fixed point theorem in a certain Sobolev space, by using a finite element approximation and constructive error estimates. We then prove the local uniqueness *separately* for eigenvalues and eigenfunctions. This local uniqueness assures the simplicity of the eigenvalue. Numerical examples are presented.

[4] **K. Nagatou and M. T. Nakao, An enclosure method of eigenvalues for the elliptic operator linearlized at an exact solution of nonlinear problems, a special issue of Linear Algebra and its Applications on "Linear Algebra in Self-Validating Methods" 324/1-3 (2001), pp. 81-106.**
We consider eigenvalue enclosing for the elliptic operator which is the linearization at an exact solution of some nonlinear elliptic equation. This problem is important in the mathematically rigorous analysis of stability or bifurcation properties of some solutions to nonlinear problems. We formulate such kind of eigenvalue problem as a nonlinear system which contains both the linearized eigenvalue problem and the original nonlinear equation. We also control the indices of

eigenvalues, and thus especially we can consider the first eigenvalue of such a problem. In these enclosing procedures, finite dimensional verified computations for linear and nonlinear systems of equations play an essential role. A numerical example is presented.

[5] **K. Nagatou, M. T. Nakao and M. Wakayama, Verified numerical computations for eigenvalues of non-commutative harmonic oscillators, Numerical Functional Analysis and Optimization, 23(5 & 6) (2002), pp. 633-650.**
We study the eigenvalue problem for an operator which defines a sort of non-trivial coupling of usual harmonic oscillators. Since one has only a limited understanding of its eigenvalues, of the behavior of eigenfunctions, and in particular of the multiplicity of eigenvalues, here we try to make a numerical approach to this system. More precisely, applying a numerical enclosure method for elliptic eigenvalue problems which is based on a verification procedure for nonlinear elliptic equations, adapted to such coupled-type eigenvalue problems on an unbounded domain, we develop a verified numerical computation for eigenvalues which also gives guaranteed information about multiplicity.

[6] **K. Nagatou, Mathematical principle for eigenvalue problems (Japanese title: "Koyuchi mondai no suri"), Bulletin of the Japan Society for Industrial and Applied Mathematics, Vol.13 No.3 (2003), pp. 58-71 (in Japanese).**
Eigenvalues of linear operators take an important role to understand a nonlinear phenomenon in science and engineering. Especially, it often becomes a key value when we consider a behaviour of dynamical systems. This paper concerns with eigenvalue problems for matricies and differential operators. We first deal with a theoretical framework for it by introducing a concept of spectrum, and consider matrix eigenvalue problems. Then we concern with infinite dimensional eigenvalue problems to obtain eigenvalues and eigenvectors for an operator in infinite dimensional domain of definition. We explain it from classical procedure to our recent results based on computer assisted proofs. We also deal with some applications of the enclosed eigenvalues and eigenvectors to algebla, another numerical verification method for nonlinear problems and stability analysis of bifurcation phenomenon in hydrodinamics.

[7] **K. Nagatou, A computer-assisted proof on the stability of the Kolmogorov flows of incompressible viscous fluid, Journal of Computational and Applied Mathematics, 169/1 (2004), pp. 33-44.**
There exists a huge number of references concerned with bifurcations and stability results for the Navier-Stokes equations. Only a few, however, provide a rigorous result which guarantees stability or instability. Our aim is to present a rigorous theorem which proves the stability of certain solutions arising in what is called the Kolmogorov problem. We accomplish this by verified computation. The eigenvalue problem arising in the Kolmogorov problem is not self-adjoint and, accordingly, it is quite difficult to treat theoretically. Our method is a rigorous numerical approach to deal with this difficulty, and numerical examples are given as a demonstration.

[8] **K. Hashimoto, K. Nagatou, M.T. Nakao, A computational approach to constructive a priori error estimate for finite element approximations of bi-harmonic problems in nonconvex polygonal domains, Information 9, No. 4 (2006), pp. 573-580.**
In numerical verification methods for solutions of nonlinear fourth order elliptic equations in nonconvex polygonal domains, it is important to find explicitly the crucial constant in constructive a priori error estimates for the finite element approximation of bi-harmonic problems. We

construct such procedures by verified computational techniques, using Hermite spline functions, for a two dimensional L-shaped domain. Several numerical examples which confirm the actual effectiveness of the method are presented.

[9] **K. Nagatou, K. Hashimoto and M. T. Nakao, Numerical verification of stationary solutions for Navier-Stokes problems, Journal of Computational and Applied Mathematics, 199 (2007), pp. 445-451.**
We present a numerical method to enclose stationary solutions of the Navier-Stokes equations, especially of the 2-D driven cavity problem with regularized boundary condition. Our method is based on an infinite dimensional Newton method, which in particular needs bounds for the inverse of the corresponding linearized operator. The method can be applied to problems with high Reynolds numbers, and we show some numerical examples which confirm the actual effectiveness.

[10] **K. Nagatou, Validated computation for infinite dimensional eigenvalue problems, 12th GAMM - IMACS International Symposium on Scientific Computing, Computer Arithmetic and Validated Numerics, Duisburg, 26 - 29 September 2006, IEEE Computer Society (Order Number E2821, 2007).**
We show how guaranteed bounds for eigenvalues (together with eigenvectors) are obtained and how non-existence of eigenvalues in a concrete region can be assured. Some examples for several types of operators in bounded and unbounded domains are presented. Furthermore we discuss possible future applications to eigenvalue enclosing/excluding of Schrödinger operators, hopefully even in spectral gaps.

[11] **M. T. Nakao, K. Hashimoto and K. Nagatou, A computational approach to constructive a priori and a posteriori error estimates for Bi-Harmonic problems, GAKUTO International Series, Mathematical Sciences and Applications, 28 (2008), pp. 139-148.**
In numerical verification methods for solutions of nonlinear fourth order elliptic equations, it is important to find the crucial constants in constructive a priori and a posteriori error estimates for finite element approximations to bi-harmonic problems. We develop procedures to accomplish this problem by verified computational techniques using Hermite spline functions for two dimensional rectangular domains. Several numerical examples which confirm the actual effectiveness of the method are presented.

[12] **K. Nagatou, Validated computations for fundamental solutions of linear ordinary differential operators, International Series of Numerical Mathematics, Vol. 157 (2008), pp. 43-50.**
We present a method to enclose fundamental solutions of linear ordinary differential equations, especially for a one dimensional Schrödinger equation which has a periodic potential. Our method is based on Floquet theory and our verification method for nonlinear equations. We show how to enclose fundamental solutions together with characteristic exponents and give a numerical example.

[13] **K. Nagatou, Numerical verification method for infinite dimensional eigenvalue problems, Japan Journal of Industrial and Applied Mathematics 26-2/3 (2009), pp. 477-491.**
We consider an eigenvalue problem for differential operators, and show how guaranteed bounds

for eigenvalues (together with eigenvectors) are obtained and how non-existence of eigenvalues in a concrete region can be assured. Some examples for several types of operators are presented.

[14] **A. Gilányi, K. Nagatou, P. Volkmann, On the stability of a functional equation characterizing the absolute value of additive functions, Annals of functional Analysis 1, No.2 (2010), pp.1-6.**
We prove the stability of a functional equation for real valued functions defined on a square-symmetric groupoid with a left unit element, and of an equation for real valued functions defined on an Abelian group.

[15] **Y. Watanabe, K. Nagatou, M. Plum, M. T. Nakao, A computer-assisted stability proof for the Orr-Sommerfeld problem with Poiseuille flow, a special issue of "Nonlinear Theory and Its Applications, IEICE" on "Recent Progress in Verified Numerical Computations", Vol.2, No.1 (2011), pp. 123-127.**
This paper describes a computer-assisted stability proof for the Orr-Sommerfeld problem with Poiseuille flow. It is an application of a numerical verification technique for second-order elliptic boundary value problems introduced by a part of the authors.

[16] **K. Nagatou, T. Morifuji, An Enclosure Method for Complex Eigenvalues of Ordinary Differential Operators, a special issue of "Nonlinear Theory and Its Applications, IEICE" on "Recent Progress in Verified Numerical Computations", Vol.2, No.1 (2011), pp. 111-122.**
An enclosure method for complex eigenvalues is presented. We formulate the eigenvalue problem as a nonlinear system and apply a fixed point theorem to enclose eigenvalues and eigenfunctions or basis of the corresponding invariant subspaces in case of multiple eigenvalues. Some enclosure examples are given.

[17] **K. Nagatou, M. Plum, M.T. Nakao, Eigenvalue excluding for perturbed-periodic one-dimensional Schrödinger operators, Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, vol. 468, issue 2138 (2012), pp. 545-562.**
Subject of investigation in this paper is a 1D-Schrödinger equation, where the potential is a sum of a periodic function and a perturbation decaying at $\pm\infty$. It is well known that the essential spectrum consists of spectral bands, and that there may or may not be additional eigenvalues below the lowest band or in the gaps between the bands. While *enclosures* for gap eigenvalues can comparably easily be obtained from numerical approximation, e.g. by D. Weinstein's bounds, there seems to be no method available so far which is able to *exclude* eigenvalues in spectral gaps, i.e. which identifies sub-regions (of a gap) which contain no eigenvalues. Here, we propose such a method. It makes heavy use of computer assistance; nevertheless, the results are completely rigorous in the strict mathematical sense, since all computational errors are taken into account.

[18] **S. Cai, K. Nagatou, Y. Watanabe, A numerical verification method for a system of FitzHugh-Nagumo type, Numerical Functional Analysis and Optimization 33 (10) (2012), pp. 1195-1220.**
We propose a numerical method to enclose a solution of FitzHugh-Nagumo equation with Neumann boundary conditions. We construct, on the computer, a set which satisfies the hypothesis

of Schauder's fixed point theorem for a compact map in a certain Sobolev space, and which therefore contains a solution. Several verified results are presented.

[19] **T. Dohnal, K. Nagatou, M. Plum, W. Reichel, Interfaces Supporting Surface Gap Soliton Ground States in the 1D Nonlinear Schrödinger Equation, Journal of Mathematical Analysis and Applications 407 (2013), pp. 425-435.**
We consider the problem of verifying the existence of H^1 ground states of the 1D nonlinear Schrödinger equation for an interface of two periodic structures:

$$-u'' + V(x)u - \lambda u = \Gamma(x)|u|^{p-1}u \quad \text{on } \mathbb{R}$$

with $V(x) = V_1(x), \Gamma(x) = \Gamma_1(x)$ for $x \geq 0$ and $V(x) = V_2(x), \Gamma(x) = \Gamma_2(x)$ for $x < 0$. Here $V_1, V_2, \Gamma_1, \Gamma_2$ are periodic, $\lambda < \min \sigma(-\frac{d^2}{dx^2} + V)$, and $p > 1$. The article [T. Dohnal, M. Plum and W. Reichel, "Surface Gap Soliton Ground States for the Nonlinear Schrödinger Equation," *Comm. Math. Phys.* **308**, 511-542 (2011)] provides in the 1D case an existence criterion in the form of an integral inequality involving the linear potentials V_1, V_2 and the Bloch waves of the operators $-\frac{d^2}{dx^2} + V_{1,2} - \lambda$. We choose here the classes of piecewise constant and piecewise linear potentials $V_{1,2}$ and check this criterion for a set of parameter values. In the piecewise constant case the Bloch waves are calculated explicitly and in the piecewise linear case verified enclosures of the Bloch waves are computed numerically. The integrals in the criterion are evaluated via interval arithmetic so that rigorous existence statements are produced. Examples of interfaces supporting ground states are reported including such, for which ground state existence follows for all periodic $\Gamma_{1,2}$ with $\operatorname{ess\,sup} \Gamma_{1,2} > 0$.

[20] **Y. Watanabe, K. Nagatou, M. Plum, M.T. Nakao, Verified computations of eigenvalue exclosures for eigenvalue problems in Hilbert spaces, SIAM Journal on Numerical Analysis, Vol. 52, No. 2 (May 2014) pp. 975-992.**
This paper presents eigenvalue excluding methods for self-adjoint or non-self-adjoint eigenvalue problems in Hilbert spaces, including problems with partial differential operators. Eigenvalue exclosure means the determination of subsets of the complex field which do not contain eigenvalues of the given problem. Several verified eigenvalue excluding results for ordinary and partial differential operators are reported on.

[21] **Y. Watanabe, K. Nagatou, M. Plum, M.T. Nakao, Norm bound computation for inverses of linear operators in Hilbert spaces, Journal of Differential Equations, Vol. 260, Issue 7, 5 April 2016, pp. 6363-6374.**
This paper presents a computer-assisted procedure to prove the invertibility of a linear operator which is the sum of an unbounded bijective and a bounded operator in a Hilbert space, and to compute a bound for the norm of its inverse. By using some projection and constructive a priori error estimates, the invertibility condition together with the norm computation is formulated as an inequality based upon a method originally developed by the authors for obtaining existence and enclosure results for nonlinear partial differential equations. Several examples which confirm the actual effectiveness of the procedure are reported.

[22] **Y. Watanabe, M.T. Nakao, K. Nagatou, On the compactness of a nonlinear operator related to stream function-vorticity formulation for the Navier-Stokes equations, JSIAM Letters Vol.9 (2017) pp.77-80.**
A compactness proof of a nonlinear operator related to stream function-vorticity formulation

for the Navier-Stokes equations is presented. The compactness of the operator provides important information for fixed-point formulations, especially for computer-assisted proofs based on Schauder's fixed-point theorem. Our idea for the compactness proof comes from books by Girault & Raviart and Ladyzhenskaia, and our principle would be also applied to convex polygonal regions.

[23] **Marius Eckert, Kaori Nagatou, Felix Rey, Oliver Stark, Sören Hohmann, Controllability and Energy Optimal Control of Time-Variant Fractional Systems, 57th IEEE Conference on Decision and Control (CDC 2018), Miami, FL, December 17-19, 2018, IEEE, Piscataway (NJ).**
Time-variant fractional models are used to describe many applications, e.g. lithium-ion batteries. For such models, neither a controllability criterion for state space equations nor the energy-optimal control function are available so far. To overcome this limitation, in this paper a reachability and controllability definition for time-variant fractional state space systems is formulated and the analytical solution of the energy-optimal control problem is derived. In this context, a fractional Gramian-Matrix appears. The fact that the fractional system state must be interpreted differently than the state of a regular system is central to this paper. The reason for the difference in the interpretation is that the fractional system state additionally depends on an initialization procedure.

[24] **Marius Eckert, Kaori Nagatou, Felix Rey, Oliver Stark, Sören Hohmann, Solution of Time-Variant Fractional Differential Equations With a Generalized PeanoBaker Series, IEEE Control Systems Letters, 3 (1), 79-84 (2019).**
Time-variant fractional systems have many applications. For example, they can be used for system identification of lithium-ion batteries. However, the analytical solution of the time-variant fractional pseudo state space equation is missing so far. To overcome this limitation, this letter introduces a novel matrix approach, namely the generalized PeanoBaker series, which is comparable to the transition matrix in the case of ordinary systems. Using this matrix, the solution of the time-variant fractional pseudo state space equation is derived. The initialization process is taken into account, which has been proven to be a crucial part for fractional operator calculus. Following this initialization, a modified definition of a fractional pseudo state is presented.

[25] **K. Nagatou, M. Plum, P. J. McKenna, Orbital stability investigations for travelling waves in a nonlinearly supported beam, Journal of Differential Equations 268(1), pp. 80-114 (2019).**
We consider the fourth-order wave equation $\varphi_{tt} + \varphi_{xxxx} + f(\varphi) = 0$, $(x,t) \in \mathbb{R} \times \mathbb{R}$ with a nonlinearity f vanishing at 0. Traveling waves $\varphi(x,t) = u(x-ct)$ satisfy the ODE $u'''' + c^2u'' + f(u) = 0$ on $\mathbb{R}$, and for the case $f(u) = e^u - 1$, the existence of at least 36 solitary travelling waves was proved in [Breuer et al, Journal of Differential Equations (2016)] by computer assisted means. We investigate the orbital stability of these solutions via computation of their Morse indices and using results from [Grillakis et al, Journal of Functional Analysis (1987 & 1990)]. In order to achieve our results we make use of both analytical and computer-assisted techniques.

Proceedings

(1) N. Yamamoto, K. Nagatou, M. T. Nakao, Numerical verification method for elliptic eigenvalue

problems and its applications (in Japanese), Keisankikagaku-kenkyu-houkoku, Research Institute for Information Technology, Kyushu University, Vol 14 (1997), pp. 53-60.

(2) K. Nagatou, M. T. Nakao, M. Wakayama, Verified computations for eigenvalues of non-commutative harmonic oscillators, Kokyuroku, Research Institute for Mathematical Sciences, Kyoto University , Vol. 1147 (2000), pp. 88-97.

(3) K. Nagatou, An enclosure method for real eigenvalues of non-selfadjoint elliptic operators, Proceedings of the Fifth China-Japan Joint Seminar on Numerical Mathematics, held in Shanghai Jiaotong University, China, August 21-25, 2000 (Z. Shi, H. Kawarada, eds.), Science Press, Beijing, China (2002), pp. 57-64.

(4) K. Nagatou, State of the Art for the Numerical Verification Method of Infinite Dimensional Eigenvalue Problems, Kokyuroku, Research Institute for Mathematical Sciences, Kyoto University, Vol.1320 (2003), pp. 121-130.

(5) M.T. Nakao, K. Nagatou and K. Hashimoto, Numerical enclosure of solutions for two dimensional driven cavity problems, (327.pdf p.1-10), Proceedings of European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2004), Jyvaskyla, Finland, 24-28 July 2004. Edited by P. Neittaanmaki, T. Rossi, K. Majava, O. Pironneau (eds.) and O. Nevanlinna, R. Rannacher (assoc. eds.).

(6) M. T. Nakao, K. Nagatou, K. Hashimoto, Numerical verification by infinite dimensional Newton's method for stationary solutions of the Navier-Stokes problems, Kokyuroku, Research Institute for Mathematical Sciences, Kyoto University, Vol.1441 (2005), pp. 61-74.

(7) K. Nagatou, Numerical verification method for spectral problems, Kokyuroku, Research Institute for Mathematical Sciences, Kyoto University, Vol.1607 (2008), pp. 13-31.