Karlsruhe Institute of Technology

Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 2021

Speaker: Dr. Björn de Rijk
July 23, 2021, 14:00 - 15:30
Zoom Link: https://kit-lecture.zoom.us/j/7143665630
Meeting ID: 714 366 5630

Nonlinear stability of periodic waves in the Lugiato-Lefever equation against localized perturbations

Institut für Analysis, Universität Stuttgart

Abstract

The Lugiato-Lefever equation is a damped nonlinear Schrödinger equation with forcing that arises in nonlinear optics. Recently, it has received considerable attention as a model for high-frequency combs generated by microresonators in periodic optical waveguides. One of the mathematical questions raised by the physical problem concerns the stability of its periodic solutions. So far, nonlinear stability of such solutions has only been established against co-periodic perturbations by exploiting the existence of a spectral gap. In this talk, we establish nonlinear stability against localized perturbations. In this case the spectrum of the linearization about the periodic wave is entirely continuous and necessarily touches the imaginary axis at the origin thanks to translational invariance of the model. Thus, there is no spectral gap and we rely on a substantially different method with origins in the stability analysis of periodic waves in reaction-diffusion systems. The main idea is to decompose the semigroup generated by the linearization in Bloch frequency space and introduce a spatio-temporal phase modulation to capture the most critical dynamics. However, in contrast to the reaction-diffusion case, the quasilinear iteration scheme for the modulated perturbation does not close due to an inherent loss of derivatives. We present a new iteration scheme incorporating the unmodulated perturbation, which satisfies a semilinear equation in which no derivatives are lost, yet where decay is too slow to close an independent iteration scheme.

This is joint work with Mathew Johnson, Mariana Haragus and Wesley Perkins.