Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 2021

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L^p-extrapolation of non-local operators

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In this talk, we discuss non-local operators like elliptic integrodifferential operators of fractional type

$$Au := \text{p.v.} \int_{\mathbb{R}^d} \frac{u(x) - u(y)}{|x-y|^{d+2\alpha}} \, \mathrm{d}y \tag{1}$$

or the Stokes operator with bounded measurable coefficients μ, formally given by

$$Au := -\operatorname{div}(\mu \nabla u) + \nabla \phi, \quad \operatorname{div}(u) = 0 \quad \text{in } \mathbb{R}^d. \tag{2}$$

These operators satisfy L^2-resolvent estimates of the form

$$\|\lambda(\lambda + A)^{-1} f\|_{\mathrm{L}^2} \leq C \|f\|_{\mathrm{L}^2} \qquad (f \in \mathrm{L}^2(\mathbb{R}^d))$$

for λ in some complex sector $\{z \in \mathbb{C} \setminus \{0\} : |\arg(z)| < \theta\}$. We describe how analogues of such a resolvent estimate can be established in L^p by virtue of certain non-local Caccioppoli inequalities. Such estimates build the foundation for many important functional analytic properties of these operators like maximal L^q-regularity.

More precisely, we establish resolvent estimates in L^p for p satisfying

$$\Big| \frac{1}{p} - \frac{1}{2} \Big| < \frac{\alpha}{d}$$

in the case (1) and

$$\Big| \frac{1}{p} - \frac{1}{2} \Big| < \frac{1}{d} \tag{3}$$

in the case (2). This resembles a well-known situation for elliptic systems in divergence form with L^∞-coefficients. Here, important estimates like Gaussian upper bounds for the semigroup cease to exist and the L^p-extrapolation has be concluded by other means. In particular, for elliptic systems one can establish resolvent bounds for numbers p that satisfy (3) and if $d \geq 3$, Davies constructed examples which show that corresponding resolvent bounds do not hold for numbers $1 < p < \infty$ that satisfy

$$\Big| \frac{1}{p} - \frac{1}{2} \Big| > \frac{1}{d}.$$

These elliptic results give an indication that the result for the Stokes operator with L^∞-coefficients is optimal as well.