

Validity of the Whitham Approximation for a Complex Cubic Klein-Gordon Equation

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Abstract

The complex cubic Klein-Gordon (ccKG) equation possesses a family of periodic traveling wave solutions. Whitham's modulation equations (WME) can be derived by a multiple scaling perturbation analysis in order to describe slow modulations in time and space of these traveling wave solutions. We prove estimates between true solutions of the ccKG equation and their associated WME approximation. The bounds are obtained in Gevrey spaces and hold independently of the spectral stability of the underlying traveling wave solutions. The proof is based on a suitable choice of variables, Cauchy-Kovalevskaya theory, infinitely many near identity changes of variables, and energy estimates in Gevrey spaces. The analysis for the ccKG equation is more complicated than the analysis for the nonlinear Schrödinger (NLS) equation which has been handled in the existing literature, due to additional curves of eigenvalues leading to an additional oscillatory behavior. This talk is based on a joint work with Xian Liao and Guido Schneider.