Karlsruhe Institute of Technology

Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 24

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Seminar room: SR 3.069

Two Dimensional Very Weak Solutions to the Monge-Ampère Equation in $C^{1,\frac{1}{3}-}$

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Abstract

In this informal seminar, I would like to discuss a recent convex integration result on the very weak solutions to the two-dimensional Monge-Ampère equation i.e. we are looking for a function $v\colon \Omega \to \mathbb{R}$, $\Omega \subset \mathbb{R}^2$ be a simply connected domain being a solution to

$$\mathcal{D}et D^2 v = f \text{ in } \Omega \,,$$

where $\mathcal{D}et D^2$ is the distribution, denoted as *very weak Hessian* introduced by Iwaniec, and given by

$$\mathcal{D}et D^2 v = \partial^2_{12}\left(\partial_1 v \partial_2 v\right) - \frac{1}{2}\partial^2_{22}\left((\partial_1 v)^2\right) - \frac{1}{2}\partial^2_{11}\left((\partial_2 v)^2\right) = -\frac{1}{2}\operatorname{curl}\operatorname{curl}(\nabla v \otimes \nabla v)\,.$$

In fact for any $\theta < \frac{1}{3}$ very weak solutions to the two-dimensional Monge-Ampère equation with regularity $C^{1,\theta}$ are dense in the space of continuous functions.

To achieve $\theta < \frac{1}{3}$ a subtle decomposition of the defect at each stage is needed. The decomposition diagonalizes the defect and, in addition, incorporates some of the leading-order error terms of the first perturbation, effectively reducing the required amount of perturbations to one.

I aim to explain why in the setting of the very weak Monge-Ampère equation and of isometric embeddings it is important to reduce the number of spirals/perturbations in each stage as much as possible. If time permits I would like to give an idea of why the above-mentioned diagonalizations allow us to reduce it in the case of Monge-Ampère essentially to one.

This is joint work with Wentao Cao and Dominik Inauen.